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Ching-Yang Lin
International University of Japan

Hiroaki Miyamoto
International University of Japan

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Ching-Yang Lin
International University of Japan

Hiroaki Miyamoto*
International University of Japan

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Abstract

This paper studies how well a simple search and matching model can describe aggregate Japanese labor market dynamics in a full information setting. We develop a discrete-time search and matching model with a convex vacancy posting cost and three shocks: productivity, separation, and mark-up shocks. We use the model as a data-generating process for our empirical analysis and estimate it by using Bayesian methods. The model is successful in replicating the behavior of unemployment and vacancies in Japan. However, we also find that the success of the model relies on shock processes that are not empirically plausible.

JEL Classification: C11; C51; E24; J64

Keywords: Search and matching model; Unemployment; Bayesian Estimation; Japanese labor market

*Corresponding Address: Hiroaki Miyamoto, hmiyamot@iuj.ac.jp, 777 Kokusai-cho, Minami Uonuma-shi, Niigata 949-7277 JAPAN, TEL., +81-25-779-1464, FAX., +81-25-779-1187. This paper was written during my stay in the Center for International Research on the Japanese Economy (CIRJE) at the University of Tokyo. I am very grateful for their hospitality. Part of this research is supported by the Grants-in-Aid for Young Scientists of the Japan Society for the Promotion of Science (Kakenhi No. 23730197 and No. 24730179).
1 Introduction

The search and matching model has been often used for studying aggregate labor markets. However, the model has recently criticized for its inability to account for the cyclical properties of the U.S. labor market. Shimer (2005) demonstrates that the model cannot generate the observed unemployment and vacancy fluctuations in response to productivity shocks of reasonable size. This failure of the model has come to be known as the “Shimer puzzle”.¹ Recently, a number of papers study whether the Shimer puzzle holds for the Japanese labor market (Esteban-Pretel et al., 2011; Miyamoto, 2011; Tawara, 2011).² In order to examine whether the model is able to capture the data, these studies use the calibration method and concentrate on the model’s ability to replicate a few key statistics. One issue with such an approach is that information on some parameters in the model is difficult to pin down.³ Furthermore, it is hard to study the quantitative implications of the entire model.

The purpose of the paper is to study how well the search and matching model can describe aggregate Japanese labor market dynamics in a full information setting. We treat our model as a data-generating process for aggregate labor market variables, and estimate a set of key parameters that drive cyclical labor market dynamics. We also examine the source and size of fluctuations and evaluate the ability of the search and matching model to replicate cyclical behaviors of the Japanese labor market.

We develop a simple discrete-time search and matching model with a convex vacancy posting cost and three shocks: productivity, separation, and mark-up shocks. We incorporate the convex vacancy posting cost since it is known that the curvature of the vacancy posting cost affects the quantitative property of the search and matching model (Fujita and Ramey, 2007; Yahiv, 2006). Incorporating a persistent shock to the separation rate is motivated by the fact that the unemployment inflow rate significantly contributes the unemployment dynamics in Japan (Miyamoto, 2011; Lin and Miyamoto, 2012)⁴. Besides these two extensions, we assume that firms produce differentiated prod-

¹In the literature, many solutions have been proposed to solve this problem. See Hornstein, Krusell, and Violante (2005) and Nagypál and Mortensen (2007) for surveys.
²While the methodology to answer the question is different among these studies, all papers reach the same conclusion that the Shimer puzzle holds for the Japanese economy.
³As Lubik (2009) mentioned, calibrating the search and matching model tends to be problematic since some of the model parameters, such as the flow value of unemployment and the worker’s bargaining power, are difficult to pin down.
⁴Recent empirical studies demonstrate that both unemployment inflow and outflow rates significantly contribute the unemployment dynamics in Japan. Miyamoto (2011) and Lin and Miyamoto (2012) ex-
ucts in a monopolistically competitive market, unlike the standard search and matching model. This modification allows us to analyze an effect of mark-up variations on labor market dynamics, as in Rotemberg (2008) and Lubik (2009).

We first ignore the mark-up shock and estimate the model using Bayesian methods for data on unemployment and vacancies in Japan. While model parameters are chosen to match selected data moments in calibration methods, they are selected by taking into account all moments of the data in our structural estimation. The structural estimation of the full model allows us to examine the ability of the model as a plausible description of labor market dynamics. We find that parameters are tightly estimated and shifted away from their priors, indicating the data are informative and parameters are identified. In order to match the data, the model estimates requires a high replacement ratio and a low worker’s bargaining power. These parameter estimates are consistent with what Hagedorn and Manovskii (2008) suggest in their calibration.

We also find that the model is capable of replicating the behavior of unemployment and vacancies remarkably well. Specifically, the model replicates the volatility of unemployment and vacancies and a negative relationship between them (the Beveridge curve) in the data. Given that the model parameters are estimated to match the data, in general, this is not surprising. However, it is well known that search and matching models cannot generate the observed negative relationship between unemployment and vacancies when the separation rate is counter-cyclically moving in the model (Fujita and Ramey, 2012). Thus, our finding is important since our model can replicate the Beveridge curve even when the separation rate moves counter-cyclically.

We find that the relative success of the model in replicating the cyclical behavior of the labor market relies on shock processes that are not empirically plausible. The volatility of labor productivity process inferred through the estimation process is too large relative to that of the observed labor productivity series. On the other hand, the volatility of separation process in the estimated model is too small relative to that of the observed separation rate series. Under these shocks, volatilities of unemployment and vacancies relative to the volatility of output are too small in the estimated model. This is another manifestation of the Shimer puzzle.

We also explore the implication of wage rigidity and the mark-up shock. We find that a sluggish wage determination mechanism and the mark-up shock help improving the ability of the model to fit the data. However, the model still faces the problem

amine the relative importance of inflow and outflow rates for fluctuations in unemployment, and find approximately a 50:50 inflow/outflow split to unemployment variation in Japan.
that the results of the model rely on atypical shock processes that are not empirically plausible. This finding casts doubt on the viability of the search and matching model to provide a theory of labor market dynamics, as suggested by Lubik (2009).

This study is related to the recent literature on the quantitative implications of the search and matching model. A number of papers study the ability of the search and matching model to account for the cyclical properties of the Japanese labor market (Esteban-Pretel et al., 2011; Miyamoto, 2011; Tawara, 2011). While they use the calibration methods and concentrate on the model’s ability to replicate a few key statistics, we rather study the quantitative implications of the entire search and matching model by using Bayesian estimation. This paper reaches the conclusion that the Shimer puzzle holds for the Japanese economy as previous studies obtained. With our best knowledge, this is the first structural empirical study of the search and matching model for Japan.

This paper is closely related to Lubik (2009, 2011). He estimates a search and matching model using Bayesian methods for the U.S. data and the Hong-Kong data. He demonstrates that basically a search and matching model is successful in describing the US and Hong-Kong labor markets well. However, Lubik (2009) shows that the success of the model relies on atypical shock process that may not have economic justification. By using the Japanese data, we show that the search and matching model succeeds to explain the dynamics of unemployment and vacancies well. However, we show that the model still fails to generate the observed volatilities in labor market variables relative to output. This finding complements the results of Lubik (2009, 2011).

The remainder of the paper is organized as follows. Section 2 presents salient features of the Japanese labor market over the business-cycle. Section 3 develops a simple discrete-time search and matching model with a convex vacancy posting cost and three shocks: productivity, separation, and mark-up shocks. We use this model as a data-generating process for our empirical analysis. In Section 4, we discuss the data and priors used for our estimation. We then present the estimation results and discuss the model’s ability to fit the data and sources of labor market fluctuations. In Section 5, we examine the quantitative performance of the search model by changing model specification and choices of observables and shocks. Section 6 concludes.

2 Cyclical properties of the Japanese labor market

We now present the cyclical characteristics of the Japanese labor market. We are mainly interested in cyclical behavior of labor productivity $A$, and five labor market variables:
the unemployment rate $u$, the vacancy rate $v$, the job finding rate $f$, the separation rate $s$, and the real wage $w$.

Table 1: Summary statistics, quarterly Japanese data, 1980-2009

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$f$</th>
<th>$s$</th>
<th>$w$</th>
<th>$y$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.061</td>
<td>0.095</td>
<td>0.081</td>
<td>0.087</td>
<td>0.010</td>
<td>0.015</td>
<td>0.013</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.825</td>
<td>0.920</td>
<td>-0.107</td>
<td>0.235</td>
<td>0.592</td>
<td>0.764</td>
<td>0.659</td>
</tr>
<tr>
<td>Correlation matrix</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td>1</td>
<td>-0.804</td>
<td>-0.468</td>
<td>0.537</td>
<td>-0.451</td>
<td>-0.735</td>
<td>-0.572</td>
</tr>
<tr>
<td>$v$</td>
<td>-</td>
<td>1</td>
<td>0.405</td>
<td>-0.577</td>
<td>0.628</td>
<td>0.765</td>
<td>0.666</td>
</tr>
<tr>
<td>$f$</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.458</td>
<td>0.220</td>
<td>0.293</td>
<td>0.207</td>
</tr>
<tr>
<td>$s$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.384</td>
<td>-0.515</td>
<td>-0.484</td>
</tr>
<tr>
<td>$w$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0.547</td>
<td>0.501</td>
</tr>
<tr>
<td>$y$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0.952</td>
</tr>
<tr>
<td>$A$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: The unemployment rate $u$ is constructed from the LFS. The vacancy rate $v$ is constructed from Employment Security Service Statistics. The job-finding rate $f$ and the separation rate $s$ are constructed from the LFS. See the text for data construction details. $u$, $v$, $f$, and $s$ are quarterly averages of monthly series. Real wages $w$ are taken from MLS. The output series $y$ is real GDP in per capita term. Labor productivity $A$ is measured as real output per employed workers. I seasonally adjust all series using the Census’s X-12-ARIMA algorithm. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600. Sample covers 1980Q1-2009Q4.

Labor productivity is measured as real output per employed workers. The output comes from the National Income and Product Accounts, while employment is obtained from the Labour Force Survey (LFS) conducted by the Statistics Bureau and the Director-General for Policy Planning. We obtain the unemployment rate from the LFS. The vacancy rate is obtained from the monthly Report on Employment Service (Shokugyo Antei Gyomu Tokei) conducted by the Ministry of Health, Labour and Welfare (MHLW). Following Miyamoto (2011) and Lin and Miyamoto (2012), we construct the job finding and separation rates from the LFS. Real wages are taken from Monthly Labour Survey (MLS) conducted by the MHLW. We are also interested in the cyclical behavior of output $y$. The output series is real gross domestic product in per capita term. All data are

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5We define the job finding rate as the rate of transition from unemployment to employment, and the separation rate as the rate of transition from employment to unemployment.
Figure 1: cyclical behavior of indicators in labor market

Note: The dashed line indicates the cyclical component of labor productivity. The solid lines indicate the cyclical components of the unemployment rate, the vacancy rate, the job-finding rate, and the separation rate. All series are reported in logs as deviations from the HP trend with smoothing parameter 1600. Sample covers 1980Q1-2009Q4.
seasonally adjusted by using the Census Bureau’s X12 filter. To focus on cyclical fluctuations, we de-trend the logged data using Hodrick-Prescott (HP) filter with smoothing parameter 1,600. The sample covers the period 1980Q1-2009Q4.

Figure 1 displays the cyclical components of variables of interest and Table 1 summarizes key statistical moments. Figure 1 (a)-(b) and Table 1 show that the unemployment rate is counter-cyclical and the vacancy rate is pro-cyclical. The correlation between the unemployment rate and labor productivity is -0.57. The correlation between the vacancy rate and labor productivity is 0.67. Since the unemployment rate is counter-cyclical and the vacancy rate is procyclical, these two series co-move negatively. The correlation between them is -0.80. The negative correlation between unemployment and vacancy, known as the Beveridge curve, can be also observed in Figure 1.

Both the unemployment rate and the vacancy rate are more volatile than labor productivity. While the standard deviation of labor productivity is 1.3 percent, the standard deviations of the unemployment and vacancy rates are 6.1 percent and 9.5 percent, respectively. Thus, the unemployment rate is about 4.5 times more volatile than labor productivity and the vacancy rate is about 7 times more volatile than labor productivity. The unemployment and vacancy rates exhibit large persistence with autocorrelation of 0.83 and 0.92, respectively.

Figure 1 (c) and (d) show cyclical components of the job finding and separation rates. The job finding rate is pro-cyclical and the separation rate is counter-cyclical. The correlation between the job finding rate and labor productivity is 0.21. The correlation between the separation rate and labor productivity is -0.48. The standard deviations of job finding and separation rates are 8.1 percent and 8.7 percent, respectively. Similar to unemployment and vacancy rates, both job finding and separation rates fluctuates much more than labor productivity. Volatilities of these two series are roughly six times as large as that of labor productivity. The autocorrelation of the job finding rate is -0.11, and that of the separation rate is 0.23.

The real wage is pro-cyclical. The correlation between the real wage and labor productivity is 0.5. The standard deviation of the real wage is 1 percent, and thus the real wage fluctuates as much as labor productivity. The cyclical behavior of output is similar to that of labor productivity. The correlation between output and labor productivity is 0.95.
3 The model

We develop a discrete time search and matching model that we use as a data-generating process for the empirical analysis. The basic structure of the model follows Pissarides (2000). In order to capture the importance of an unemployment inflow channel in generating unemployment fluctuations, we incorporate a persistent shock to a separation rate. We also introduce a convex vacancy posting cost since it is known that the curvature of the vacancy posting cost affects the quantitative property of the search and matching model.\(^6\) Beside these two extensions, we assume that firms produce differentiated products in a monopolistically competitive market, unlike the standard search and matching model. We incorporate this modification to analyze the effect of mark-up variations on labor market dynamics, as in Rotemberg (2008) and Lubik (2009).

The environment There are three types of agents in the economy: workers, firms, and retailers. Workers are identical, risk neutral and live forever. The number of total workers is normalized to one. A worker can be either employed or unemployed. If a worker is employed, he produces output and earns wages. If a worker is unemployed, he gets a flow utility from non-market activity and searches for a job.

Firms are identical, risk neutral and live forever. Firms recruit workers by posting vacancies. When firms employ workers, they produce wholesale goods and sell them to retailers in a competitive market. Firms maximize their intertemporal profit functions by choosing the number of vacancies to be posted. Workers are separated from firms at a stochastic exogenous rate.

Retailers buy wholesale goods and differentiate them to produce final goods. Final goods are sold in a monopolistic competitive market. The final goods are assumed to be numeraire.

The labor market The labor market is subject to frictions and firms and workers cannot meet instantaneously but must go through a time-consuming search process. The number of successful job matches is determined by the Cobb-Douglas matching function,

\[ m(u_t, v_t) = m_0u_\alpha v_t^{1-\alpha}, \]

\(^6\)It is known that a property of vacancy posting costs affects a quantitative property of a search and matching model. The vacancy behavior in the search and matching model is analyzed, for example, by Fujita and Ramey (2007) who introduce the fixed cost for creating a new job opening and Yahiv (2007) who introduces the curvature in vacancy posting costs as in our model.
where $u_t$ is the number of unemployed workers, $v_t$ is the number of vacancies, $m_0$ represents match efficiency and $0 < \alpha < 1$ is the elasticity of the matching function with respect to unemployment. Define $\theta_t \equiv v_t / u_t$, as labor market tightness. The probability of a firm with a vacancy is matched with a worker is $m(u_t, v_t) / v_t = m_0 \theta_t^{1-\alpha} \equiv q(\theta_t)$. Similarly, the probability that an unemployed worker is matched is $m(u_t, v_t) / u_t = m_0 \theta_t^{1-\alpha} = \theta_t q(\theta_t)$. The number of employed workers is defined as
\[ n_t = 1 - u_t. \] (1)

Matches are destroyed at an exogenous separation rate $s_t$, which takes place at the end of period $t$.

We assume that it takes one period for new matches to be productive and that both old and new matches face the same separation rate. The evolution of employed workers is given by
\[ n_t = (1 - s_t) \left[ n_{t-1} + m u_{t-1} \theta_{t-1}^{1-\alpha} v_{t-1} \right]. \] (2)
Thus, the number of employed workers at time $t$ is given by the number of employed workers at time $t - 1$ plus new matches formed in period $t$ that were not destroyed.

**Retailer’s pricing rule**  Workers consume final goods produced by retailers. Instead of formally modeling a worker’s consumption choice problem, we rather assume that workers have Dixit-Stiglitz preferences over a continuum of final goods. Thus, the pricing rule in the retail sector is given by
\[ p_t = \frac{\epsilon_t}{\epsilon_t + 1} p^w_t, \]
where $p_t$ is the price of final goods, $p^w_t$ is the price of wholesale goods, and $\epsilon_t$ is the elasticity of demand. Denote the price mark-up $\epsilon_t / (\epsilon_t + 1)$ as $M_t$. Then, the relative wholesale goods price $p^w_t / p_t$ is given by $1 / M_t$.

**Firm’s optimization**  Production takes place when a firm is matched with workers. Output $y_t$ of a typical firm is linear in employment $n_t$:
\[ y_t = A_t n_t, \]
where $A_t$ is an aggregate productivity.

In order to hire workers, firms have to post vacancies. The cost of posting vacancies is $\kappa v_t^\phi / \phi$ where $\kappa > 0$ and $\phi > 0$.\footnote{In the standard search and matching model, the cost of posting vacancies is assumed to be linear.} A firm pays workers real wages $w_t$, which will be
derived below. The firm chooses the optimal number of vacancies $v_t$ to be posted and its employment level $n_t$ by maximizing the following intertemporal profit function

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{A_t n_t}{M_t} - w_t n_t - \frac{\kappa}{\phi} v_t^\phi \right]$$

subject to the equation of employment evolution (2). The first-order conditions are

$$\mu_t = \frac{A_t}{M_t} - w_t + \beta \mathbb{E}_t [\mu_{t+1} (1 - s_{t+1})],$$

$$\frac{\kappa v_t^{\phi-1}}{q(\theta_t)} = \beta \mathbb{E}_t [\mu_{t+1} (1 - s_{t+1})],$$

where $\mu_t$ is the Lagrangian multiplier on constraint (2).

Making use of these two first-order conditions, we can obtain the job creation condition

$$\frac{\kappa v_t^{\phi-1}}{q(\theta_t)} = \beta \mathbb{E}_t \left[ \left( \frac{A_{t+1}}{M_{t+1}} - w_{t+1} + \frac{\kappa v_{t+1}^{\phi-1}}{q(\theta_{t+1})} \right) (1 - s_{t+1}) \right].$$

(3)

The job creation condition states that expected cost of posting a vacancy, the left-hand side of (3), is equal to the firm’s share of the expected new surplus from a new job match, the right-hand side of (3).

**Wage determination**  Wages are determined by Nash bargaining between a firm and a worker, where the worker has bargaining power $\eta \in (0, 1)$. The surplus sharing implies

$$(1 - \eta) (W_t - U_t) = \eta J_t,$$

(4)

where $W_t$ is the value of an employed worker, $U_t$ is the value of an unemployed worker, and $J_t$ is the value of the marginal value of the worker to the firm.

The value of an employed worker is characterized by the following Bellman equation:

$$W_t = w_t + \beta \mathbb{E}_t [(1 - s_{t+1}) W_{t+1} + s_{t+1} U_{t+1}].$$

(5)

The value of $W_t$ is determined by several factors. In the current period, the worker receives wage $w_t$. In the next period, while the worker retains his job with probability $1 - s_{t+1}$, he loses the job and becomes unemployed with probability $s_{t+1}$.

The value of an unemployed worker is

$$U_t = b + \beta \mathbb{E}_t [\theta_t q(\theta_t) (1 - s_{t+1}) W_{t+1} + (1 - \theta_t q(\theta_t)(1 - s_{t+1})) U_{t+1}].$$

(6)
An unemployed worker receives unemployment benefits \( b \) and transitions into employment with probability \( \theta_t q(\theta_t) (1 - s_{t+1}) \).

The value of the marginal worker to the firm \( J_t \) is equivalent to the Lagrangian multiplier on employment constraint \( \mu_t \), which is the shadow value of a filled job. Thus, the value of a filled job is

\[
J_t = \frac{A_t}{M_t} - w_t + \beta \mathbb{E}_t \left[ J_{t+1} (1 - s_{t+1}) \right]. \tag{7}
\]

By using the wage sharing rule (4), value functions (5), (6), (7), and the first-order conditions, we obtain the following wage equation

\[
w_t = \eta \frac{A_t}{M_t} + (1 - \eta) b + \eta \theta_t \kappa v_i^{\phi-1}. \tag{8}
\]

By substituting (8) into (3), the job creation condition (3) can be rewritten as

\[
\frac{\kappa v_i^{\phi-1}}{q(\theta_t)} = \beta \mathbb{E}_t (1 - s_{t+1}) \left[ (1 - \eta) \left( \frac{A_{t+1}}{M_{t+1}} - b \right) - \eta \theta_{t+1} \kappa v_{i+1}^{\phi-1} + \frac{\kappa v_i^{\phi-1}}{q(\theta_{t+1})} \right]. \tag{9}
\]

The dynamics of the model are given by the three equations (1), (2), and (9) and the definition of the labor market tightness that solve for four unknowns \( n_t, u_t, v_t, \) and \( \theta_t \).

### Shocks

There are three shocks in this model economy: the productivity shock \( A_t \), the separation rate shock \( s_t \), and the mark-up shock \( e_t \). We assume that these shocks follow first-order autoregressive processes of the form:

\[
\log X_t = (1 - \rho_X) \log X + \rho_X \log X_t + \varepsilon_{X,t},
\]

where \( 0 < \rho_X < 1, \varepsilon_{X,t} \sim N(0, \sigma_X^2) \) and \( X_t = \{ A_t, s_t, e_t \} \). Since labor productivity and the separation rate are negatively correlated in the data, we assume that the innovations of \( A_t \) and \( s_t \) are correlated. The correlation between two series is denoted by \( \rho_{A,s} \).

### 4 Estimation

The model is estimated by using Bayesian methods. First, we log-linearize the non-linear model around a deterministic steady state. We then solve the model and apply the Kalman filter to evaluate the likelihood function of the observable variables. The
likelihood function and the prior distribution of the parameters in the model are combined to obtain the posterior distribution. The posterior kernel is simulated numerically by employing the random-walk Metropolis-Hastings algorithm.\(^8\)

To estimate the model, there should be at least as many sources of uncertainty in the empirical model as there are observables. For the benchmark specification, we use observations on two series: the unemployment rate \(u\) and the vacancy rate \(v\). We use these two series as observables since they are central variables in the search and matching model. Among three stochastic processes incorporated in the model, the productivity shock \(A_t\) and the separation shock \(s_t\) are selected as the exogenous unobservable stochastic process. Thus, the benchmark model does not allow for variations in the mark-up.

There are several reasons why we choose productivity and separation shocks as sources of variation in the empirical model. First, they are the fundamental exogenous shocks in the search and matching literature, while the mark-up shock has been criticized as non-structural shock in some studies (see for example, Chari et al., 2008). Second, since we obtain the data on labor productivity and the separation rate, we can further evaluate the performance of the model by comparing the estimated shock processes with the observed data. In an extension of the benchmark specification, we add the series of output \(y\) and wages \(w\) and the mark-up shock.\(^9\)

In the following, we first discuss the selection of the prior distributions. We then report the estimated parameters and evaluate the model’s performance.

### 4.1 Prior distribution and parameters

In the baseline estimation, we use observations on the unemployment rate and the vacancy rate. The sample covers 1980Q1-2009Q4. We obtain these two series from the LFS and the monthly Report on Employment Service conducted by MHLW (please see Section 2 for the detail). All data are seasonally adjusted and de-trended using the HP filter with smoothing parameter 1,600.

The model contains 9 structural parameters, excluding shock parameters. We choose priors for the Bayesian estimation based on the typical values in calibration studies. We set the discount rate \(\beta = 0.99\) because the annual real interest rate has been around 4%.

\(^8\)Details on the estimation procedure can be found in An and Schorfheide (2007) and Lubik (2009).

\(^9\)Although we have an information about stochastic processes of \(A_t\) and \(s_t\), it cannot be used in the estimation directly. The reason is that the information of exogenous shocks cannot help to identify parameters in the model structure, but parameters of shock processes.
Elasticity of demand $\epsilon$ is set to be $-10$. This implies that the mark-up is 1.1, a conventional value in the literature. The remaining parameters are estimated. We use Beta distributions for parameters that take sensible values between zero and one, Gamma distributions for real-valued parameters, and the inverse Gamma distributions for the shock variances.

The prior of the matching constant $m_0$ is chosen to be consistent with the observed job-finding rate of 0.142 per month (Miyamoto, 2011). Kano and Ohta (2002) estimate the matching function in the Japanese labor market by using aggregate data, and obtain the elasticity of the matching function $\alpha$ of about 0.6. We set $\alpha$ at a mean of 0.6 with a standard deviation of 0.15. This leads to a prior mean of $m_0 = 0.15$. The prior mean of the separation rate $s$ is set to 0.012, as estimated by Miyamoto (2011).

We now choose priors for the unemployment benefit $b$ and the worker’s bargaining power $\eta$. $^{10}$ These two parameters have been the subject of some discussion in the literature. Martin (1998) computes the average replacement rates, the ratio of unemployment benefits to average wages, in the OECD countries and reports that the replacement rate in Japan is about 0.6. We set $b$ at a mean of 0.6 with a wide coverage region. Regarding the worker’s bargaining power, since we are interested in how much information on $\eta$ is in the data, we choose a uniform prior over the unit interval.

Regarding parameters in the vacancy cost function, the prior mean of the vacancy posting elasticity $\phi$ is set equal to 1 with a large standard deviation. A linear vacancy cost is the standard assumption in the literature. Then, the scale parameter is set to $\kappa = 0.45$, which is obtained from the steady-state solutions of the model.

The prior mean of the autoregressive parameters is set equal to 0.5 and the prior mean of the standard errors is set equal to 0.01 for all shocks. They are typical values used in the literature. The priors are summarized in columns 3-5 in Table 2.

$^{10}$Much of the debate on the viability of the search and matching model as a description of the labor market centers around these parameter values. Shimer (2005) sets $b$ by targeting the replacement ratio of 0.4. Hagedorn and Manovskii (2008) argue that Shimer’s choice of the value of the opportunity cost of employment is too low because it does not allow for the value of leisure, home production, or unemployment benefits. They calibrate the opportunity cost of employment and the worker’s bargaining power to match the observed cyclical response of wages and average profit rate. Their results are $b = 0.955$ and $\eta = 0.052$. Mortensen and Nagyp9 (2007) criticize Hagedorn and Manovskii (2008) for using these parameters because these parameters yield workers a gain of 2.8% in flow utility by going from unemployment to employment.
### Table 2: Parameter estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Distribution</th>
<th>Density</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Mean</th>
<th>90 Percent Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>Fixed</td>
<td>0.99</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of demand</td>
<td>Fixed</td>
<td>-10</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Match elasticity</td>
<td>Beta</td>
<td>0.60</td>
<td>0.15</td>
<td>0.598</td>
<td>[0.549, 0.644]</td>
<td></td>
</tr>
<tr>
<td>$m_0$</td>
<td>Match efficiency</td>
<td>Gamma</td>
<td>0.15</td>
<td>0.05</td>
<td>0.217</td>
<td>[0.161, 0.272]</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>Separation rate</td>
<td>Beta</td>
<td>0.012</td>
<td>0.002</td>
<td>0.011</td>
<td>[0.008, 0.014]</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>Worker’s bargaining power</td>
<td>Uniform</td>
<td>0.50</td>
<td>0.25</td>
<td>0.376</td>
<td>[0.001, 0.743]</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>Unemployment benefit</td>
<td>Beta</td>
<td>0.60</td>
<td>0.2</td>
<td>0.861</td>
<td>[0.707, 0.994]</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>Elasticity of vacancy creation</td>
<td>Gamma</td>
<td>1.0</td>
<td>0.50</td>
<td>3.627</td>
<td>[2.749, 4.532]</td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Scaling factor on vacancy creation</td>
<td>Gamma</td>
<td>0.45</td>
<td>0.10</td>
<td>0.459</td>
<td>[0.307, 0.617]</td>
<td></td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>AR-coefficients of shocks</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.934</td>
<td>[0.895, 0.973]</td>
<td></td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>AR-coefficients of shocks</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.144</td>
<td>[0.018, 0.266]</td>
<td></td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Standard deviation of shocks</td>
<td>Inverse Gamma</td>
<td>0.01</td>
<td>1.00</td>
<td>0.035</td>
<td>[0.005, 0.070]</td>
<td></td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Standard deviation of shocks</td>
<td>Inverse Gamma</td>
<td>0.01</td>
<td>1.00</td>
<td>0.038</td>
<td>[0.025, 0.051]</td>
<td></td>
</tr>
<tr>
<td>$\rho_{A,s}$</td>
<td>Correlation between shocks</td>
<td>Uniform</td>
<td>-0.5</td>
<td>0.29</td>
<td>-0.155</td>
<td>[-0.301, 0.000]</td>
<td></td>
</tr>
</tbody>
</table>

### 4.2 Posterior estimates of the parameters

Table 2 (columns 6-7) reports posterior means together with their 90 percent confidence intervals. Figure 2 shows the prior and posterior distributions. In general, the parameter means and distributions are moved considerably from their priors, indicating the data are informative about the values of estimated parameters.

We begin by seeing the worker’s bargaining power $\eta$ and the unemployment benefit $b$. The posterior means of the worker’s bargaining power and the unemployment benefit are 0.376 and 0.861, respectively. They are moved away considerably from the priors. These parameter estimates are in favor of Hagedorn and Manovskii (2008)’s calibrated parameter values.\(^{11}\)

The posterior mean of the vacancy posting elasticity $\phi = 3.627$ is considerably shifted away from the prior. The estimated $\phi$ suggests that vacancy creation is more costly to the firm because marginal vacancy posting costs are increasing in the number

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\(^{11}\)In Hagedorn and Manovskii (2008), the value of an unemployment benefit is high and the value of a worker’s bargaining power is low.
Figure 2: Prior and posterior distribution
of vacancies. This estimate is substantially different from what is typically assumed in the calibration literature. In most studies, vacancy costs are assumed to be linear, i.e. $\phi = 1$. The high value of $\phi$ may be interpreted as a balancing factor that mitigates excessive vacancy creation due to the low worker’s bargaining power.

The estimates of the scale parameter $\kappa$ and the separation rate $s$ are not identified in a purely econometric sense, since their posterior distributions overlap with the priors. This finding is consistent with Lubik (2009, 2011). The posterior mean of the match elasticity $\alpha$ is close to their prior means. The posterior mean of the match elasticity $\alpha = 0.598$ is in the plausible range of 0.5-0.7 reported by Petrongolo and Pissarides (2001).

| Case | Baseline $\phi = 1$ | Wage Rigidity | $u, v, w$ | $u, v, y$ | $u, y$ | $v, y$ | $u, w$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.598</td>
<td>0.594</td>
<td>0.596</td>
<td>0.585</td>
<td>0.595</td>
<td>0.495</td>
<td>0.699</td>
</tr>
<tr>
<td>$m_0$</td>
<td>0.217</td>
<td>0.231</td>
<td>0.223</td>
<td>0.228</td>
<td>0.249</td>
<td>0.189</td>
<td>0.180</td>
</tr>
<tr>
<td>$s$</td>
<td>0.011</td>
<td>0.012</td>
<td>0.012</td>
<td>0.014</td>
<td>0.013</td>
<td>0.012</td>
<td>0.010</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.376</td>
<td>0.034</td>
<td>0.576</td>
<td>0.108</td>
<td>0.510</td>
<td>0.221</td>
<td>0.185</td>
</tr>
<tr>
<td>$b$</td>
<td>0.861</td>
<td>0.595</td>
<td>0.798</td>
<td>0.985</td>
<td>0.897</td>
<td>0.694</td>
<td>0.342</td>
</tr>
<tr>
<td>$\phi$</td>
<td>3.627</td>
<td>-</td>
<td>3.841</td>
<td>3.233</td>
<td>4.151</td>
<td>1.344</td>
<td>1.052</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.459</td>
<td>0.422</td>
<td>0.383</td>
<td>0.495</td>
<td>-</td>
<td>0.522</td>
<td>0.476</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.934</td>
<td>0.935</td>
<td>0.931</td>
<td>0.944</td>
<td>0.756</td>
<td>0.648</td>
<td>0.757</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.144</td>
<td>0.129</td>
<td>0.148</td>
<td>0.107</td>
<td>0.189</td>
<td>0.271</td>
<td>0.764</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.035</td>
<td>0.042</td>
<td>0.012</td>
<td>0.005</td>
<td>0.010</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>0.038</td>
<td>0.037</td>
<td>0.038</td>
<td>0.045</td>
<td>0.033</td>
<td>0.114</td>
<td>0.079</td>
</tr>
<tr>
<td>$\rho_{A,s}$</td>
<td>-0.155</td>
<td>-0.071</td>
<td>-0.143</td>
<td>-0.222</td>
<td>-0.080</td>
<td>-0.184</td>
<td>-0.018</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-</td>
<td>-</td>
<td>0.359</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{c}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.390</td>
<td>0.927</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{c}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.736</td>
<td>0.361</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 4.3 The model evaluation

We now examine the quantitative performance of the benchmark model along several dimensions. We assess the ability of the model to capture the observed behavior of unemployment and vacancies. We also assess the model’s predictions for unobserved
variables, such as output and wages. Furthermore, we are interested in the sources of business cycle. Specifically, we examine how much productivity and separation shocks contribute to movements in unemployment and vacancies, and whether these underlying shocks are consistent with what we observed in the data.

Table 4: Data and model fit

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline</th>
<th>$\phi = 1$</th>
<th>Wage Rigidity</th>
<th>$u, v, w$</th>
<th>$u, v, y$</th>
<th>$u, y$</th>
<th>$v, y$</th>
<th>$u, w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal Likelihood</td>
<td>474.276</td>
<td>429.036</td>
<td>475.635</td>
<td>876.342</td>
<td>846.391</td>
<td>616.250</td>
<td>634.861</td>
<td>643.587</td>
<td></td>
</tr>
<tr>
<td>$\sigma(u)$</td>
<td>0.061</td>
<td>0.073</td>
<td>0.070</td>
<td>0.060</td>
<td>0.074</td>
<td>0.064</td>
<td>0.073</td>
<td>0.093</td>
<td>0.071</td>
</tr>
<tr>
<td>$\sigma(v)$</td>
<td>0.095</td>
<td>0.102</td>
<td>0.109</td>
<td>0.077</td>
<td>0.097</td>
<td>0.089</td>
<td>0.046</td>
<td>0.085</td>
<td>0.032</td>
</tr>
<tr>
<td>$\sigma(w)$</td>
<td>0.010</td>
<td>0.083</td>
<td>0.046</td>
<td>0.023</td>
<td>0.012</td>
<td>0.015</td>
<td>0.006</td>
<td>0.006</td>
<td>0.011</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>0.015</td>
<td>0.099</td>
<td>0.118</td>
<td>0.033</td>
<td>0.017</td>
<td>0.015</td>
<td>0.014</td>
<td>0.016</td>
<td>0.015</td>
</tr>
<tr>
<td>$\sigma(a)$</td>
<td>0.013</td>
<td>0.098</td>
<td>0.117</td>
<td>0.032</td>
<td>0.015</td>
<td>0.015</td>
<td>0.013</td>
<td>0.016</td>
<td>0.015</td>
</tr>
<tr>
<td>$\sigma(s)$</td>
<td>0.087</td>
<td>0.039</td>
<td>0.037</td>
<td>0.038</td>
<td>0.046</td>
<td>0.034</td>
<td>0.119</td>
<td>0.122</td>
<td>0.107</td>
</tr>
<tr>
<td>$\rho(u, v)$</td>
<td>-0.800</td>
<td>-0.850</td>
<td>-0.800</td>
<td>-0.800</td>
<td>-0.840</td>
<td>-0.810</td>
<td>0.820</td>
<td>0.940</td>
<td>0.880</td>
</tr>
<tr>
<td>$\rho(a, s)$</td>
<td>-0.480</td>
<td>-0.060</td>
<td>-0.030</td>
<td>-0.060</td>
<td>-0.080</td>
<td>-0.060</td>
<td>-0.160</td>
<td>-0.020</td>
<td>-0.270</td>
</tr>
<tr>
<td>$\sigma(u)/\sigma(y)$</td>
<td>4.69</td>
<td>0.74</td>
<td>0.59</td>
<td>1.83</td>
<td>4.41</td>
<td>4.21</td>
<td>5.43</td>
<td>5.73</td>
<td>4.61</td>
</tr>
<tr>
<td>$\sigma(v)/\sigma(y)$</td>
<td>7.31</td>
<td>1.03</td>
<td>0.92</td>
<td>2.33</td>
<td>5.79</td>
<td>5.84</td>
<td>3.39</td>
<td>5.22</td>
<td>2.07</td>
</tr>
<tr>
<td>$\sigma(w)/\sigma(y)$</td>
<td>0.67</td>
<td>0.84</td>
<td>0.39</td>
<td>0.69</td>
<td>0.69</td>
<td>0.98</td>
<td>0.41</td>
<td>0.40</td>
<td>0.69</td>
</tr>
</tbody>
</table>

In order to see how well the model fits the data, we compute various statistics from simulation of the estimated model with parameters set at their posterior means. Column (2) of Table 4 summarizes the main results from the model. For the comparison, counterpart data moments are reported in Column (1) of Table 4.

The model accounts for the volatility of the unemployment rate and the vacancy rate remarkably well. In the data, the standard deviations of the unemployment rate and the vacancy rate are 0.061 and 0.095, respectively. The corresponding values implied by the model are 0.073 and 0.102, respectively. This result implies that the search and matching model is a good data-generating process for the unemployment and vacancy rates.

In the calibration literature, the relative standard deviations of labor market variables to output are used to evaluate the quantitative performance of the search and matching model. The relative standard deviations of unemployment and vacancies to output in the data are 4.69 and 7.31, respectively. The corresponding values implied by the model are 0.74 and 1.03, respectively. This is because the output volatility is way
too large. While the standard deviation of output is 0.015 in the data, it is 0.099 in the model. This is another manifestation of the Shimer puzzle in the sense that the volatilities of unemployment and vacancies relative to output volatility are too small. This means that the Shimer puzzle still holds under the parameter values supported by the full information of the data set. Furthermore, the relative volatility of wages to output predicted by the model is larger than that in the data. This suggests the necessity of a source of wage rigidity to capture the empirical wage pattern.

The model is successful in capturing a strong negative correlation between unemployment and vacancies, i.e. the Beveridge curve. One may think that this result is not surprising since we estimate parameters to fit the observed unemployment and vacancy processes. However, it is well-known that search and matching models cannot generate the observed strong negative correlation between unemployment and vacancies when the separation rate is counter-cyclically moving in the model (see Fujita and Ramey (2012)). Thus, our finding is important since our model replicates the Beveridge curve even when the separation rate fluctuates. We will discuss this issue later.

Table 5: Variance decompositions

<table>
<thead>
<tr>
<th>Case</th>
<th>(1) Baseline</th>
<th>(2) φ = 1</th>
<th>(3) Wage Rigidity</th>
<th>(4) u, v, w</th>
<th>(5) u, v, y</th>
<th>(6) u, y</th>
<th>(7) v, y</th>
<th>(8) u, w</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[a, s]</td>
<td>[a, s]</td>
<td>[a, s]</td>
<td>[a, s, ε]</td>
<td>[a, s, ε]</td>
<td>[a, s]</td>
<td>[a, s]</td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>[79.5, 20.5]</td>
<td>[84.4, 15.6]</td>
<td>[68.8, 31.2]</td>
<td>[78.6, 21.3, 0.1]</td>
<td>[1.5, 25.2, 273.3]</td>
<td>[4.8, 95.2]</td>
<td>[0.1, 99.9]</td>
<td>[8.9, 91.1]</td>
</tr>
<tr>
<td>v</td>
<td>[99.5, 0.5]</td>
<td>[93.6, 6.4]</td>
<td>[99.3, 0.7]</td>
<td>[99.0, 0.6, 0.4]</td>
<td>[1.3, 0.5, 98.2]</td>
<td>[2.9, 97.1]</td>
<td>[0.3, 99.7]</td>
<td>[6.2, 93.8]</td>
</tr>
<tr>
<td>y</td>
<td>[100.0, 0.0]</td>
<td>[100.0, 0.0]</td>
<td>[100.0, 0.0]</td>
<td>[99.7, 0.3, 0.0]</td>
<td>[99.5, 0.1, 1.0]</td>
<td>[97.0, 3.0]</td>
<td>[96.2, 3.8]</td>
<td>[98.0, 2.0]</td>
</tr>
<tr>
<td>w</td>
<td>[100.0, 0.0]</td>
<td>[100.0, 0.0]</td>
<td>[100.0, 0.0]</td>
<td>[48.1, 0.0, 51.9]</td>
<td>[59.8, 0.4, 39.8]</td>
<td>[79.8, 20.2]</td>
<td>[86.1, 13.9]</td>
<td>[95.1, 4.9]</td>
</tr>
</tbody>
</table>

We now compute variance decompositions to study the sources of business-cycle. Specifically, we are interested in how much productivity and separation rate shocks contribute to movements in unemployment and vacancies. The results are reported in Column (1) of Table 5. Both shocks play an important role, but the productivity shock contributes the volatility in unemployment and vacancies more. The productivity shock explains 79.5 percent of the volatility in the unemployment rate and 99 percent in the vacancy rate. The separation shock is relatively more important in driving unemployment than vacancies since it takes the role of a residual in the employment equation (2).

Our finding that unemployment and vacancies are mainly driven by the productivity shock in the estimated model can be interpreted as follows. In order to fit the
observed cyclical patterns of unemployment and vacancies, the model requires a large productivity shock as the dominant source of fluctuations. This point is also suggested by Shimer (2005). However, an important question is now raised: Are the exogenous shocks that drive unemployment and vacancy fluctuations in the estimated model really the sources of observed fluctuations in unemployment and vacancies?

To match the observed behavior of unemployment and vacancies, the model requires particular stochastic processes $\hat{A}_t$ and $\hat{s}_t$. These two series are treated as un-observables when the model is estimated. However, we can obtain the series on labor productivity and the separation rate from the data. If shock processes implied by estimation are inconsistent with the observed data, one may still doubt the model’s ability to explaining cyclical behavior of the labor market variables, even though the model’s predictions on series of $u$ and $v$ are satisfactory.

We now examine if the estimated shock processes are empirically consistent with observed data by comparing the cyclical properties of $A_t$ and $s_t$ in the data with those in the model. Results are reported in Columns (1)-(2) in Table 4. While the volatility of $A_t$ in the model is about nine time as large as one in the data, the volatility of $s_t$ in the model is about half of one in the data. Furthermore, the correlation between $A_t$ and $s_t$ in the model is much weaker than one in the data. These results are consistent with our earlier finding that the model requires a large productivity shock as the dominant source of fluctuations to capture the observed patterns of $u$ and $v$. However such large volatile $A_t$ also induces large fluctuations in $y_t$, which is not consistent with what we observe in the data. The discrepancy of $A_t$ and $s_t$ between in the model and in the data casts doubt on the ability of the benchmark model as description of labor market dynamics. This implies that in order to fix this problem, either some adjustment should be made in the model setting, or some other exogenous sources of shocks should be considered. We delve further into this issue in the next section.

Figure 3 plots impulse response functions of unemployment and vacancies to positive one-standard deviation shocks to productivity and the separation rate. They are computed at the posterior means of the structural parameters. On impact, a positive productivity shock leads to an increase in vacancies. A higher productivity level encourages firms open more vacancies since it increases the expected return to hire a worker. Furthermore, the low worker’s bargaining power implies that firms get large part of the match surplus, leading to more vacancy creation. This leads to a persistent response of

\[^{12}\text{It is important to note that in the calibration literature, the shock processes are obtained directly from the observed series and examine if the model is capable of matching unemployment and vacancy volatilities.}\]
vacancy creation. Due to the timing assumption of the matching process, on the impact of the shock, unemployment does not respond and falls in the following periods.

Next, we consider the response of unemployment and vacancies to a separation shock. On impact, unemployment jumps up and then declines in the following periods. An increase in unemployed workers increases the number of job seekers. As a result, firms have higher incentives to create more vacancies. Thus, on the impact of a positive shock on the separation rate, vacancies increase and then decline in the following period since the number of unemployed workers fall.

5 Discussion

In this section, we examine the quantitative performance of the search and matching model by changing model specifications and the choice of observables and shocks. We first explore the implications of non-linear vacancy costs and wage rigidity. We then assess the robustness of parameter estimates and the fit of the model to changes in the model specification and the choice of observables and shocks.
5.1 Model modification - linear vacancy cost $\phi = 1$

We now study the role of the curvature of the vacancy posting cost. While the vacancy cost is usually assumed to be linear in the literature, it is assumed to be convex in our model. We assess the contribution of nonlinear vacancy costs to our results by examining a model with linear vacancy costs. Thus, we set $\phi = 1$ and re-estimate the model by using the benchmark prior specification for other parameters.

The results are reported in the third column of Table 3 and Column (3) of Table 4. Since the model with linear vacancy costs and the benchmark model use the same set of observables, we evaluate overall fits of these models by comparing the marginal data densities (MDD). Table 4 shows that our benchmark specification is preferred with MDD of 474.3.

Except the worker’s bargaining power $\eta$, the posterior means are close to those in the benchmark case. The posterior mean of the bargaining parameter $\eta = 0.034$. Since the prior on this parameter is uniform, the shape of the posterior is exclusively determined by the data information. When the vacancy cost function becomes linear, it becomes less costly for a firm to open a vacancy. As a result, the volatility of $v$ is expected to be larger under the baseline parameter values. To mitigate the excess volatility of vacancies, the value of the worker’s bargaining power is expected to rise. However, the value of $\eta$ becomes much smaller than one in the baseline case. This seemingly counter-intuitive result can be understood as follows. Given a linear vacancy cost and a low worker’s bargaining power, a firm’s surplus becomes larger. This implies that shocks exercise a lower influence on firm’s decision on opening a vacancy. Thus, the low worker’s bargaining power mitigates the excess volatility of vacancies caused by the linear vacancy cost. Indeed, our numerical analysis finds that if $\eta$ becomes less than a certain level, firm’s incentive to post a vacancy becomes weaker.\textsuperscript{13}

Regarding the volatility in the labor market variables, the standard deviations of labor market variables of interest in the model with linear vacancy costs are broadly similar to those in the benchmark case with the exception of wages. The standard deviation of wages in the model with linear vacancy costs is almost half of that in the benchmark model. It is important to note that the standard deviation of output in the model with linear vacancy costs is slightly larger than that in the benchmark model. As a results, the relative standard deviations of unemployment and vacancies to output in

\textsuperscript{13}Our numerical exercise finds that a relationship between the worker’s bargaining power $\eta$ and the volatility of vacancies $\sigma_v$ is not linear. There exists a threshold $\bar{\eta}$. When $\eta > \bar{\eta}$, $\sigma_v$ increases as $\eta$ decreases. On the other hand, when $\eta < \bar{\eta}$, $\sigma_v$ decreases as $\eta$ decreases.
the model with $\phi = 1$ are smaller than those in the benchmark case.

We now examine how a curvature in a vacancy posting cost affects a relationship between unemployment and vacancies. It is well known that a search and matching model cannot generate the observed strong negative correlation between unemployment and vacancies when the separation rate is counter-cyclically moving in the model. However, our benchmark model succeeds to generate it. This could be possible because of the curvature in vacancy posting costs.\footnote{In a search and matching model with a linear vacancy posting cost, when the separation rate is counter-cyclically moving, a negative productivity shock can substantially increases the number of job seekers by increasing job separation, which in turn makes vacancy posting more attractive. As a result, the Beveridge curve is destroyed. However, the curvature in vacancy posting costs might mitigate the effect.} However, as seen in Table 4, the model with linear vacancy posting costs also generates a strong negative correlation between unemployment and vacancies. This implies that the reason that the benchmark model can successfully match the Beveridge curve is not the convex vacancy posting cost. A possible reason for it is that in our model, the volatility of the separation rate is too small relative to that of labor productivity series. Table 4 shows that the standard deviation of the separation rate is 0.039, which is less than one half of the standard deviation of labor productivity. This is not empirically plausible, because the standard deviation of the separation rate is much larger than that of labor productivity in the data.

5.2 Model modification - wage rigidity

A number of papers argue that an incorporation of wage rigidity improves the performance of a search and matching model to match the cyclical behavior of unemployment and vacancies (Hall, 2005; Shimer, 2005). We now assess whether an incorporation of wage rigidity improves the ability of our model to replicate the business cycle facts in the data.

Following Hall (2005) and Krause and Lubik (2007), we incorporate wage rigidity into our model in the form of a backward looking wage norm. Hall (2005) argues that a wage norm may arise from social convention that constrains wage adjustment. Without getting into the details of the wage norm, we assume that the actual wage is the weight average of a notional wage $w^*$ and a wage norm $w^n$:

$$w_t = \gamma w^n_t + (1 - \gamma) w^*, \quad \gamma \in [0, 1].$$

We assume that the notional wage is equal to the bargaining solution of our benchmark model and the wage norm is the wage in the steady-state of the
benchmark model. We assign the parameter $\gamma$ a Beta distribution with support on the unit interval. We set gamma at a mean of 0.5 with a wide coverage region. For other parameters, we use the prior specification in the benchmark case.

The results are reported in columns labelled as wage rigidity in Tables 3, 4, and 5. The posterior means are broadly similar to those in the benchmark model with the exception of the bargaining parameter $\eta$. The posterior mean of the worker’s bargaining power is about 1.5 times as large as one in the baseline case. The estimated coefficient of wage rigidity $\gamma$ is 0.36, with a 90% coverage interval [0.10, 0.61].

The relative standard deviations of $u$ and $v$ to output are 1.83 and 2.33, respectively. They are more than twice larger than those in the benchmark model. However, they are still much lower than the observed volatility (about 39% and 32% of the observed volatility, respectively). The standard deviation of labor productivity series inferred through estimation dramatically decreases (about 1/3 of the baseline case), but this value is still much larger than what we observe in the data.

These findings suggest that a sluggish wage determination mechanism provides a channel to amplify the effects of productivity shocks on unemployment and vacancy rates, as some recent studies suggested. However, our results show that it is not enough to solve the Shimer puzzle.

### 5.3 Estimation with different sets of observables

In the benchmark case, we use the data on unemployment and vacancies to estimate our model. We find that the model’s productivity process inferred through the estimation process is too large relative to the observed productivity series. This implies that Shimer puzzle holds although the volatilities of unemployment and vacancies in the estimated model match those in the data. This finding motivates us to consider the following hypothetical situation. Suppose that the model now could successfully serve as the data-generating process for output and the unemployment rate (i.e. the model could generate “enough” relative volatility of unemployment to output), what are parameter values supporting this hypothesis? In order to answer this question, we estimate our model by using the data on the unemployment rate and output as observables. This experiment somewhat follows the standard practice in the literature by feeding the shock process obtained from the observed series (labor productivity series) and examining if the model is capable of matching unemployment and vacancy volatiles.

The seventh column of Table 3 shows that some of estimated parameters are different from those in the benchmark model. Regarding the volatilities, although the model
implied volatilities of vacancies and wages are smaller than the data, it successfully accounts for the volatilities of output and unemployment. However, much of this success is due to the separation shock. As Table 5 shows, labor market dynamics are captured exclusively by movements in the separation rate, which takes the role of a residual in the equation determining the evolution of unemployment.

The serious shortcoming of the model is that it fails to generate a negative relationship between unemployment and vacancies. This failure can be understood by looking at variance decompositions and correlation. In the model, unemployment and the separation rate are positively correlated and vacancies and the separation rate are also positively correlated. As seen in results in variance decompositions, unemployment and vacancies are exclusively driven by the separation shock. Therefore, the model generates the positive relationship between unemployment and vacancies, although unemployment is counter-cyclical and vacancies are pro-cyclical.

We also experiment with using the sets of observations \{v, y\} and \{u, w\} in place of \{u, y\}. Some of parameter estimates are similar across specification: the matching elasticity \(\alpha\), the matching efficiency \(\xi\), the separation rate \(s\), and vacancy costs \(\kappa\). The other parameters show more variations. The models match the second moments reasonably well. However, this comes from the incidence of specific shocks. Based on the variance decomposition result, while unemployment and vacancies are mainly driven by the separation shock, output and wages are driven exclusively by the productivity shock. Thus, labor market dynamics is mainly determined by an unexplained process related to job separation. This implies that the productivity shock cannot generate empirically consistent labor market dynamics in our simple framework. Furthermore, the model fails to generate a negative relationship between unemployment and vacancies.

### 5.4 The model with a mark-up shock

So far we have estimated the model with two conventional shocks, productivity and separation shocks, by using two observables. In an extension of the baseline empirical specification, we now add a price mark-up shock. We use two sets of observables: \{u, v, w\} and \{u, v, y\}.

Results are reported in columns labelled “u, v, w” and “u, v, y” in Tables 3, 4, and 5. The parameter estimates are fairly consistent across specifications. The exception is the bargaining parameter \(\eta\). In the estimation with observables \{u, v, w\}, the bargaining parameter \(\eta\) is extremely low with the value of 0.108. For both specifications, the model matches the data and second moments reasonably well. The key difference is driving
forces of the business cycle. While unemployment and vacancies are mainly driven by the productivity shock in the estimation with \{u,v,w\}, they are mainly driven by the mark-up shock in the estimation with \{u,v,y\}.

When we compare the benchmark case and the estimation with \{u,v,w\}, we find that the parameter estimates and the driving forces of the business cycle are consistent between them. However, the model estimated using \{u,v,w\} does a better job to match second moments of the data. This is because the worker’s bargaining power is extremely low and a large part of wage fluctuation is driven by the mark-up shock in the estimation with \{u,v,w\}. This implies the model’s ability to capture the data improves once tight connection between wages and labor productivity is shut off.

Interestingly, it turns out that the model estimated using \{u,v,w\} fits the data better than the model with wage rigidity (See Columns (4) and (5) in Table 4). This difference can be understood by seeing correlation between wages and labor productively. The correlation between \(w\) and \(y\) is 1.0 in the model with wage rigidity, while it is 0.7 in the model estimated using \{u,v,w\}. Thus, the wage is still highly correlated with the productivity shock in the model with wage rigidity. In contrast, in the model estimated using \{u,v,w\}, the mark-up shock acts as a wedge between labor productivity and wages, and cuts tight connection between labor productivity and wages.

6 Conclusion

This paper studies how well a search and matching model describes aggregate Japanese labor market dynamics in a full information setting. We develop a simple search and matching model with a convex vacancy posting cost and three shocks: productivity, separation, and mark-up shocks. We use the model as a data-generating process for our empirical analysis and estimate it by using Bayesian methods. By taking into account all moments of the data and not just selected covariates, the structural estimation of the model allows us to study the ability of the model as description of labor market dynamics. To the best of our knowledge, this is the first paper to study the Japanese labor market from the perspective of the structural estimation of the search and matching model.

The model is capable of replicating the behavior of unemployment and vacancies in Japan remarkably well. Specifically, the model replicates an observed high volatility of unemployment and vacancies and a negative relationship between them. We also demonstrate that allowing for a convex vacancy cost and a sluggish wage determination
mechanism helps improving the ability of the model to fit the data. However, this paper also shows that the success of the model relies on atypical shock processes. Specifically, there exists a certain discrepancy between shock processes inferred through the estimation process and their empirical counterparts. To explore this issue remains for future research.
References


