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Abstract

How do entry and R&D subsidies affect economic growth? We build a tractable second-generation Schumpeterian growth model with endogenous markups to study the short and long run impacts of industrial policies. In particular, we contrast the effects of R&D subsidies (to vertical innovation) with the effects of entry subsidies (to horizontal innovation). We show that the subsidies affect the economy in qualitatively different fashions. The two policies have opposite implications for the number of firms: entry subsidies increase the number of firms and reduces steady-state markups, while R&D subsidies decrease the number of firms and increase steady-state markups. Furthermore, an entry subsidy increases the the economy's short-run growth rate but reduces long-run growth. In contrast, an R&D subsidy decreases the economy's short-run growth rate but increases long-run growth.

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1 Introduction

In this paper we study the connection between industrial policy and growth through the lens of an endogenous growth model with endogenous markups. Specifically, we use a second-generation growth model, which builds upon the foundations of Peretto (1996) and Peretto (1999). In this class of models, the economy's growth rate, number of firms, and markup is jointly determined and hence influenced by industrial policy. This paper sheds light on the short and long run impacts of industrial policies on the number of firms and the corresponding affects on product market competition, markups, and the economy's growth rate.

There exists a large literature in growth economics that studies the relationship between competition and economic growth. Building on the insight of the Lerner's index (a standard measure of competition), the theoretical literature has found an inverted-U shape relationship between markups and the economy's growth rate. In their work, a change in markup by definition also changes households' preferences because those two things are represented by the same parameter. Therefore, a change in markup, or competition, requires a change in households' preferences! We instead build on this literature in two ways. First, because our model features oligopolistic competition, the markup is endogenous and depends on the number of firms. Second, changes to markups are driven by changes in economic policy which affect the number of firms and hence the markup. In contrast, the existing literature relies on *exogenous* changes to the markup; in particular, changes to markups are driven by changes to the elasticity of substitution. Because changes to the elasticity of substitution—by definition—change households preferences, two distinct channels are simultaneously changed and therefore the experiment in the existing literature should be re-examined and improved. Furthermore, the aforementioned literature relies on the first-generation growth framework, which delivers the well-known counterfactual prediction that the economy's growth rate is positively related to scale.[1] In contrast, our second generation Schumpetarian framework does not suffer from the scale effect prediction.

Our model consists of two sources of innovation: vertical (quality improvement) and horizontal (variety expansion). In the vertical dimension, *oligopolistic* incumbents engage in R&D to reduce their production costs and earn higher profits. The return to R&D is affected by the market size of the firm and hence the endogenous markup. In the horizontal dimension, upon paying a sunk entry cost, entrepreneurs create new products that compete with incumbent firms. This entry affects the rate of return to R&D through two mechanisms: it segments the market and reduces markups. The former provides the mechanism which nullifies the scale effect; the ladder reduces the aggregate price level and consequently increases the market size of firms.[2]

[1] This well-known 'scale effect' has received an enormous amount of empirical attention—and has been soundly rejected (see Backus, Kehoe, and Kehoe (1992); Jones (1995)).

[2] The second-generation framework is also consistent with several empirical facts in industrial organization

As aforementioned, the majority of the literature studying 'competition' and growth study the effects of changes in markups *driven by changes in preferences.* In particular, Aghion and Howitt (1998), Bucci (2013) find an inverted-U shaped relationship between the economy's growth rate and the elasticity of substitution.[3] In our model, markups are *endogenous* and changes to them are affected by the number of firms (which drive endogenous competition). For completeness, we show that the elasticity of substitution and the economy's growth rate are negatively related.[4] The focus of this paper, however, is the relationship between endogenous competition and growth, not changes to preferences.

Governments can, and do, subsidize entry, the aim of which is to increase competition and utility through love-of-variety.[5] Governments also subsidize vertical innovation which increases the R&D expenditure of existing firms. In this class of models—and observed in the data (Sutton, 2007)—R&D expenditure per-firm and firm size are positively correlated. In other words, increased vertical innovation effort typically reduces the number of firms. Therefore, a subsidy to vertical innovation which induces more vertical innovation may *reduce* competition and hence increase markups. These two policy instruments, entry subsidies and R&D subsidies, can have opposite effects on competition.

The tractability of our framework allows us to analyze the economy's short-run and long-run response to changes in industrial policy. In particular, we show that the question "how do R&D subsidies affect the economy's growth rate?" depends on the time in question. Regarding entry subsidies, because they reduce the amount of resources devoted to final-output production, an increase in the entry subsidy leads to an immediate decrease in production. The transitional dynamics are such that the short-run growth rate increases, but long-run growth declines. It is thus possible for an entry subsidy to make an economy temporarily poorer, then temporarily richer, then permanently poorer, when compared to the counterfactual steady state. To summarize, the time path for the economy's growth rate follows an inverted-U shaped response following the introduction of an entry subsidy. The time in question will thus affect the answer to the question "is an entry subsidy good for growth?" Similarly, because it leads to the exit

literature—the first-generation framework does not capture these key features of the data. For example, both the number of firms and firms' market size are endogenous (Sutton, 2007). The return to R&D depends on firm size rather than aggregate market size (Cohen and Klepper, 1996a,b). Furthermore, productivity growth is driven mainly by in-house R&D instead of creative destruction (see Acemoglu and Cao (2015) or OECD (2003, Chapter 4) for a thorough discussion).

[3] Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and Hashmi (2013) model competition in a different, although still exogenous, way; specifically, "We model the degree of product market competition inversely by the degree to which the two firms in a neck-and-neck industry are able to collude" (Aghion, Bloom, Blundell, Griffith, and Howitt, 2005).

[4] Similarly, using a modification of Aghion and Howitt, Minniti (2010) finds a positive relationship between the elasticity of substitution and growth.

[5] Of course, governments can also affect the cost of creating a business through indirect channels (*i.e* hiring additional lawyers to be consistent with regulations).

of firms, an R&D subsidy (a subsidy to vertical innovation) can initially reduce the economy's growth rate, but lead to a higher long-run growth rate. To our knowledge, our model is the first to point out that entry and innovation subsidies affect growth differently in different time dimensions. If a policy maker's goal is to increase the economy's short-run growth rate, they can through entry subsidies. In contrast, vertical innovation subsidies decrease short-run growth, but increases long-run growth.

The rest of the paper proceeds as follows. Section 2 constructs the Schumpeterian growth model with the oligopolistic competition and endogenous market structure. Section 3 highlights interaction between product market competition and innovation and its policy implications. Section 4 concludes.

2 The Model

The framework builds on the Schumpeterian model with endogenous market structure pioneered by Peretto (1996, 1998). Labor is the sole factor of production which is used to produce: final and intermediate goods and also vertical and horizontal innovation. Intermediates are produced by *oligopolistic* firms that sell their differentiated goods to final good producers who sell their product homogenous final good to consumers. When unambiguous, we omit time subscripts.

2.1 Households

The economy is populated by a representative household with lifetime utility

$$U(t) = \int_t^\infty \log(C_s)\, e^{-\rho s} ds, \tag{1}$$

where ρ is the rate of time preference and C is the consumption. The household chooses the optimal savings plan to maximize (1) subject to its lifetime budget constraint:

$$\dot{A}_t = r_t A_t + (1-\tau) w_t - C_t, \tag{2}$$

where A is per-capita asset holdings, r_t is the real interest rate, w_t is the wage rate, and τ is the income tax rate. Applying the standard optimal control theory yields the household's Euler equation

$$\frac{\dot{C}}{C} = r - \rho, \tag{3}$$

which implies that the household's consumption is growing if the real interest rate exceeds the time preference rate at which the household discounts future consumption.

2.2 Final Goods

The final goods sector is comprised of a large number of perfectly competitive firms that produce homogeneous final output, Y, according to

$$Y = \left(\sum_0^N X_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon\beta}{\epsilon-1}} L_Y^{1-\beta}, \tag{4}$$

where $0 < \beta < 1$, $\epsilon > 1$, X_{it} is the intermediate goods, and N is the endogenous number of intermediate goods/firms. The representative firm hires labor, L_{Yt}, and purchases intermediates, X_i, to maximize its profit:

$$\Pi_Y = P_Y \left(\sum_0^N X_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon\beta}{\epsilon-1}} L_Y^{1-\beta} - wL_Y - \sum_0^N p_i X_i.$$

The price of final output is the numeraire (hence $P_Y = 1$) and p_i is the price of intermediate good i. The firm's optimization yields the demand for intermediates

$$X_i = \frac{\beta Y p_i^{-\epsilon}}{\sum_0^N p_i^{1-\epsilon}}, \tag{5}$$

and labor,

$$L_Y = \frac{(1-\beta)\, Y}{w}. \tag{6}$$

2.3 Intermediate Goods Sector

The intermediate good industry is comprised of oligopolistic incumbents. These incumbents engage in vertical R&D to reduce their variable costs of production. The number of firms, and hence number of goods is endogenously determined; upon paying a sunk entry cost, entrepreneurs create new goods. In what follows, we first discuss the determination of prices and investment decisions given the existing market structure. We then endogenize the number of firms.

2.3.1 Incumbents

The typical firm produces with the technology

$$L_{Xi} = Z_i^{-\sigma} X_i + \phi, \tag{7}$$

where L_{Xi} is the total amount of labor required to produce X_i units of good i. The costs of production consist of the variable costs, $Z_i^{-\sigma}$ (σ is the elasticity of unit costs with respect to manufacturing productivity Z_i) and fixed cost, ϕ.

By engaging in cost-reducing R&D, firms increase their knowledge stock Z_i which reduces their production costs. The R&D technology is

$$\dot{Z}_i = \eta Z L_{Zi}, \tag{8}$$

where η is R&D productivity, L_{Zi} is R&D labor hired by firm i, and Z is stock of public knowledge defined as $Z = \sum_0^N Z_i di/N$.[6]

The net-profit (gross-profits minus the fixed costs of operation and R&D expenditure) of firm intermediate producer i is

$$\pi_{Xi} = \Pi_{Xi} - w\phi - (1 - s_z)\, wL_{Zi}, \tag{9}$$

where $0 < s_Z < 1$ is an R&D subsidy and the gross profit is

$$\Pi_{Xi} = X_i \left(p_i - Z_i^{-\sigma} w\right). \tag{10}$$

The firms maximize their discounted lifetime value

$$V_i = \int_0^\infty e^{-\int_0^t [r(k)+\delta]dk} \pi_{Xit} dt, \tag{11}$$

by choosing their price and R&D outlay, subject to (5), (8), and (9). The following lemma characterizes the firms' optimal decisions:

Lemma 1. *The price of intermediate good i is*

$$p_i = wZ_i^{-\sigma} M(N), \tag{12}$$

where

$$M(N) \equiv \frac{\epsilon (N-1) + 1}{(\epsilon - 1)(N-1)}; \tag{13}$$

the rate of return to cost reduction is

$$r_Z = \left(\frac{\beta}{1-\beta}\right)\left(\frac{\eta\sigma}{1-s_Z}\right)\left(\frac{1}{M(N)}\right)\left(\frac{L_Y}{N}\right) - \delta + \frac{\dot{w}}{w} - \frac{\dot{Z}}{Z}. \tag{14}$$

[6]This specification assumes that the stock of public knowledge is the weighted sum of the firm-specific knowledge stocks. Peretto and Smulders (2002) provide the micro-foundation for this spillover function.

Proof: See the appendix.■

In addition to the wage rate, the price of intermediates is determined by firm-level productivity, Z_i, and the markup which is a function of the number of firms, $M(N)$. Because the markup is is decreasing in the number of firms,

$$\frac{dM(N)}{dN} = \frac{-1}{(\epsilon - 1)(N-1)^2} < 0, \tag{15}$$

the price of intermediates is decreasing in the number of firms. Lemma 1 also shows that the return to in-house R&D depends on L_Y/N, not solely L_Y. An increase in the number of firms segments the market, and thereby decreases the rate of return. The market segmentation, however, is offset by the reduction in the aggregate price level.

2.3.2 Entrants

Entrepreneurs hire labor to create new goods and set up firms to serve the market. The mass of firms/products evolves according to

$$\dot{N} = \psi N L_N - \delta N. \tag{16}$$

After paying a sunk entry cost, the firm enters the market with the industrial average productivity.[7] The exogenous death rate, δ, is present to avoid the asymmetric dynamics and hysteresis due to sunk entry costs. Such complications would distract from the main point of the paper. L_N is the amount of labor devoted to gross entry and ψN is the efficiency of labor at creating new products.[8] The free entry condition requires that the value of the firm equals the subsidized cost of creation; consequently, $V = (1 - s_N) w/\psi N$, where s_N is the entry subsidy.

Differentiating (11) with respect to time yields the return to entry

$$r_N = \frac{\pi}{V} + \frac{\dot{V}}{V} - \delta. \tag{17}$$

Equation (17) is the usual asset-pricing equation, indicating that the rate of return to entry is equal to the discounted flow of profits plus capital appreciation.

[7]This standard assumption preserves symmetry at all times. See Peretto (1999) for a discussion of the conditions that ensure symmetric equilibrium.

[8]See Peretto and Connolly (2007) for a discussion of alternative specifications.

2.4 Government

The government provides subsidies to R&D $s_Z \in (0,1)$ and entry $s_N \in (0,1)$. The government collects tax revenue T from the household, and the amount of tax revenue is

$$T = \tau w L = \tau (1-\beta) Y, \tag{18}$$

where $\tau \in (0,1)$ is an exogenous tax rate on wage income. The balanced-budget condition is

$$T = G + s_N w L_N + s_Z w L_Z, \tag{19}$$

where G is unproductive government spending that changes endogenously to balance the fiscal budget as in Peretto (2007). This formulation allows us to focus on the effects of subsidies by isolating the effects of taxation.

3 General Equilibrium

The equilibrium is a time path of allocations $\{A_t, C_t, Y_t, X_{it}, L_{Xit}, L_{Zit}\}$ and prices $\{r_t, w_t, p_{it}, V_t\}$, such that the following conditions hold:

- the household maximizes utility taking $\{r_t, w_t\}$ as given;
- competitive final goods firms maximize profits taking $\{w_t, p(i)\}$ as given;
- oligopolistic incumbents in the intermediate goods sector choose $\{p_{it}, L_{Zit}\}$
- entrants make entry decisions taking $\{V_t\}$ as given;
- the value of all existing oligopolistic firms adds up to the value of the household's asset such that $A_t = \sum_{i=0}^{N} V_{it}$;
- the goods market clears such that $Y_t = C_t$;
- the labor market clears such that $L = \sum_{i=0}^{N} (L_{Xit} + L_{Zit}) + L_{Yt} + L_{Nt}$;
- the asset market clears such that $r_t = r_{Zt} = r_{Nt}$.

Substituting (6), (5), (12) into (4) and imposing symmetry yields the aggregate production function:

$$Y = \left(\frac{\beta}{1-\beta}\right)^{\beta} N^{\frac{\beta}{\epsilon-1}} \left(M(N^{\star})\right)^{-\beta} Z^{\sigma\beta} L_Y, \tag{20}$$

which depends on N, Z and L_Y. Combining (6) and (3) and using that $C = Y$, yields

$$\frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{L_Y}}{L_Y} + \frac{\dot{w}}{w} = r - \rho. \tag{21}$$

The dynamics for L_Y are driven by an unstable differential equation which yields the solution characterized in our next lemma.

Lemma 2. *The labor devoted into final good production L_Y jumps to a locally determinate steady-state value:*

$$L_Y^{\star} = (1-\beta)\left[\frac{\rho\,(1-s_N)}{\psi} + (1-\tau)\,L\right]. \tag{22}$$

Proof: See the appendix. ■

It implies that the amount of labor devoted towards production (22) must be met for all time so that L_Y will jump to its long-run equilibrium value L_Y^* immediately after any shock hits the economy and will remain constant at this value along the transition path. This convenient property dramatically increases the tractability of our model.

3.1 The economy's dynamics

The growth rate of the economy's two sources of growth, variety and quality, are characterized by our first proposition.

Proposition 1. *The rate of quality growth is*

$$\frac{\dot{Z}}{Z} = \left(\frac{\eta\sigma}{(1-s_Z)\,M\,(N)}\right)\left(\frac{\beta}{1-\beta}\right)\left(\frac{L_Y^{\star}}{N}\right) - (\rho+\delta)\,, \tag{23}$$

and the entry rate is

$$\begin{aligned}\frac{\dot{N}}{N} \;=\; & \frac{\psi}{1-s_N}\left(\left(\frac{\beta}{1-\beta}\right)\frac{L_Y^{\star}\left(N\left(1-\sigma\left(\epsilon-1\right)\right)+\sigma\left(\epsilon-1\right)\right)}{N\epsilon-\left(\epsilon-1\right)} - N\left(\phi-\left(1-s_Z\right)\frac{\rho}{\eta}\right)\right)\\ & -\left(\rho+\delta\right). \end{aligned} \tag{24}$$

Proof: See the appendix. ■

Proposition 1 highlights several important points. In particular, the subsidies enter the growth rates in different—and sometimes counteracting—manners. Chiefly, it is straightforward to see that the growth rate of quality is increasing in the vertical R&D subsidy, s_Z. However, this leads to lower variety growth. The intuition, developed in Sutton's static models (Sutton, 2007) and further developed by Peretto (1996), is that the rate of return to horizontal innovation depends on net-profit: the profits minus the sunk cost (exogenous and endogenous). Because firms engage in more vertical R&D, the flow of profits is lower and consequently the incentive to enter is lower. Although s_N does not enter the growth rate of quality directly, the next section shows that the steady-state number of firms is increasing in the entry subsidy which affects the steady-state growth rate of quality.

3.2 Steady state

In this section we determine the steady-state number of firms and the growth rate. With regard to the evolution of firms, we plot the dynamics of N in figure (1).

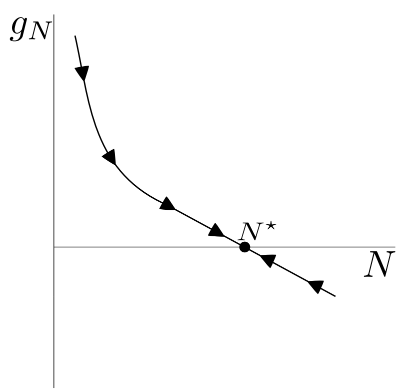


Figure 1: Dynamics of N.

In the steady state, the number of firms is constant; the steady state is obtained by setting (24) equal to zero, which leads to

$$N^\star = \underset{N}{\text{argsolve}}\left[\frac{N}{1-s_N}\left(\left(\frac{\beta L_Y^\star\left(N+\sigma\left(\epsilon-1\right)\left(1-N\right)\right)}{\left(1-\beta\right)N\left(\epsilon\left(N-1\right)+1\right)}\right)-\left(\frac{\eta\phi-\rho\left(1-s_Z\right)}{\eta}\right)\right)=\frac{\delta+\rho}{\psi}\right]. \quad (25)$$

Equation (25) yields a quadratic function

$$aN^2+bN+c=0, \quad (26)$$

where

$$a=-\psi\left(\phi-\left(1-s_z\right)\frac{\rho}{\eta}\right)\epsilon;$$

$$b=\left(\frac{\psi\beta L_Y^\star}{1-\beta}\right)\left(1-s_n-\sigma\left(\epsilon-1\right)\right)+\psi\left(\phi-\left(1-s_z\right)\frac{\rho}{\eta}\right)\left(\epsilon-1\right)-\epsilon\left(1-s_n\right)\left(\rho+\delta\right);$$

$$c=\left(\epsilon-1\right)\left[\left(\frac{\psi\beta L_Y^\star}{1-\beta}\right)\sigma+\left(1-s_n\right)\left(\rho+\delta\right)\right].$$

Our next proposition characterizes the economy's steady state.

Proposition 2. *Under the parameter restriction*

$$\eta\phi > \rho\left(1 - s_Z\right), \tag{27}$$

the number of firms converges to

$$N^\star = \frac{-b - \sqrt{b^2 - 4ac}}{2a},$$

and the steady-state growth rate is

$$g^\star \equiv \frac{\dot{Y}^\star}{Y} = \sigma\beta\left(\frac{\dot{Z}}{Z}\right)^\star = \sigma\beta\left(\frac{\eta\sigma}{\left(1 - s_Z\right)M\left(N^\star\right)}\right)\left(\frac{\beta}{1-\beta}\right)\left(\frac{L_Y^\star}{N^\star}\right) - \rho. \tag{28}$$

Proof: See the appendix.

Equation (28) demonstrates an important implication of our model. Because 'competition' means many things in the literature, for instance changes in competition have been proxied by changes in the elasticity of substation (Dixit and Stiglitz, 1977; Bucci, 2013; Minniti, 2010) or the "rate of imitation" (Grossman and Helpman, 1993; Aghion and Howitt, 1998), we must be precise. In our analysis, competition is measured not only by the elasticity of substitution but also the number of firms. The entry of new firms is particularly important in an oligopolistically competitive market. This endogenous entry affects the economy in two ways. First, it dilutes the market of existing firms. As Romer (1990) emphasized, the incentive to engage in R&D is positively related to the extent of the market. Because the firm's market size decreases, firms do less R&D and the economy's growth rate *decreases*. There is, however, a secondary mechanism that affects the growth rate in the opposite direction. The market size is also influenced by the endogenous markup. Specifically, when firms enter, the markup declines, as shown by (15). In general equilibrium, through the reduction in the aggregate price level, market size increases. This increase in the firm's market size leads to more R&D and the economy's growth rate *increases*, shown in (28). The existing literature ignores the latter channel, that competition could affect growth positively through endogenous price markup. Therefore, product market competition is not obviously good, or bad, for growth.

Given that in this class of models N is endogenous, we *cannot* engage in comparative statics by changing N. Instead, we look to policies, or other shocks to the system, that influences the steady-state number of firms; we do this in the next section. With this in mind, because of the insight of the chain-rule, it is useful to show the relationship between $N^\star$ and $g^\star$. In particular, policy changes—aimed at increasing the growth rate—will at least indirectly, if not directly, work through changes in N. The relationship is characterized by the following corollary.

Corollary 1. *When*

$$N^\star > 1 + \frac{1}{\epsilon}, \tag{29}$$

$$\frac{dg^\star}{dN^\star} < 0. \tag{30}$$

Proof: See the appendix.

There is a well-known prediction of standard Schumpetarian growth theory, because it reduces firm size, competition is bad for growth. Nonetheless, empirical work has documented an inverted-U relationship between competition and innovation.[9] The question is the markup channel—which counteracts the market dilution effect—enough to generate the observed inverted-u shape? Our answer is a resounding no. Recall that $\epsilon > 1$—consequently, for all plausible values of N, the relationship between the number of firms and the growth rate is negative. The preceding discussion does not imply that competition is bad for the economy; a larger N contributes to utility through love-of-variety and, importantly, leads to lower markups. We discuss the effects of policy changes on markups and the number of firms in the next section.

We now turn to the common experiment of analyzing the economy's response to changes in the elasticity of substitution.

Corollary 2. *When (29)*

$$\frac{dg^\star}{d\epsilon} < 0. \tag{31}$$

Proof: See the appendix.

Our model does not find an inverted-U shaped relationship between the elasticity of substitution and the economy's growth rate. An increase in ϵ decreases the markup, which through a lower price level, increases the growth rate. Moreover, the increased substitutability means that consumers value variety less and hence the steady-state number of firms declines. Both forces lead to an increase in the economy's growth rate.

4 Steady-State Comparative Statics

In this section we study the economy's dynamic response to revenue neutral changes in subsidies. We first analyze the economy's response to an increase to the entry subsidy, s_N. The following

[9] To be precise, this literature typically analyzes the relationship between innovation and ϵ, not the number of firms.

corollary establishes the relationship between the number of firms and the economy's growth rate with s_N.

Corollary 3. *Under conditions of (27) and (29), we have*

$$\frac{dN^\star}{ds_N} > 0; \tag{32}$$

$$\frac{dg^\star}{ds_N} < 0. \tag{33}$$

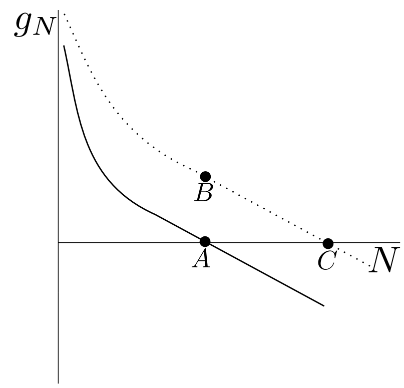
Proof: See the appendix.

Unsurprisingly, an entry subsidy leads to more firms in the steady state. However, equations (32) and (33) demonstrate a trade-off; an entry subsidy increases the number of firms, but it also reduces the economy's steady-state growth rate. We illustrate the effects of the subsidy on the steady-state number of firms in figure (2a).

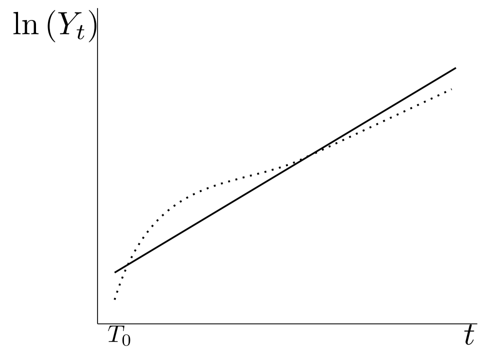
In general equilibrium, output can be written as

$$Y_t^\star = \left(\frac{\beta}{1-\beta}\right)^\beta (N^\star)^{\frac{\beta}{\epsilon-1}} (M(N^\star))^{-\beta} (Z_t^\star)^{\sigma\beta} L_Y^\star. \tag{34}$$

which demonstrates that an increase in N (induced by the entry subsidy) increases output, and hence growth. Nonetheless, because the entry subsidy reduces the steady-state growth rate of Z_t , the long-run growth rate decreases, as shown in (2b). We assume that the economy is originally in its steady-state, depicted by the solid line. We depict the affects of the policy change on the dashed line.



(a) Induced entry from an entry subsidy.



(b) Outputs response to an entry subsidy.

Figure 2: Response to an entry subsidy

Are entry subsidies good or bad for growth? The answer to this depends on the time in question. Also it is crucial to distinguish between levels and growth rates. Following an increase in the entry subsidy, output decreases. Therefore, product market competition is not obviously good, or bad, for growth—because the incentive to engage in horizontal innovation is higher, the economy devotes more resources to it and consequently output declines. However, the increased entry leads to *temporarily* faster growth rate. As depicted, the faster growth leads the economy to a *temporarily* higher income. However, as endogenous entry vanishes the effects of reduced quality growth leads us to a lower steady-state growth rate which means, in the long run, we will have lower income. In other words, the time path for the economy's growth rate exhibits an inverted-U shaped response to an entry subsidy. The time in question will thus affect the answer to the relationship between entry subsidy and economic growth.

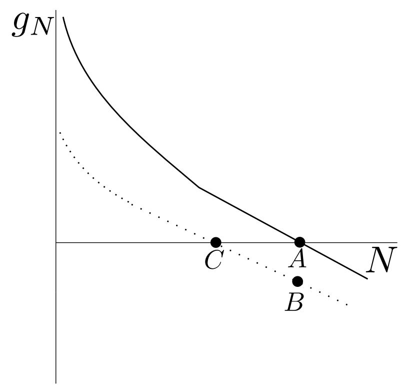


Figure 3: Exit from an R&D subsidy

We now turn to the effects of R&D subsidies. The dynamics of the number of firms is depicted in figure (3). Our next corollary characterizes the steady-state effects of R&D subsidies.

Corollary 4. *When (27) and (29) are satisfied:*

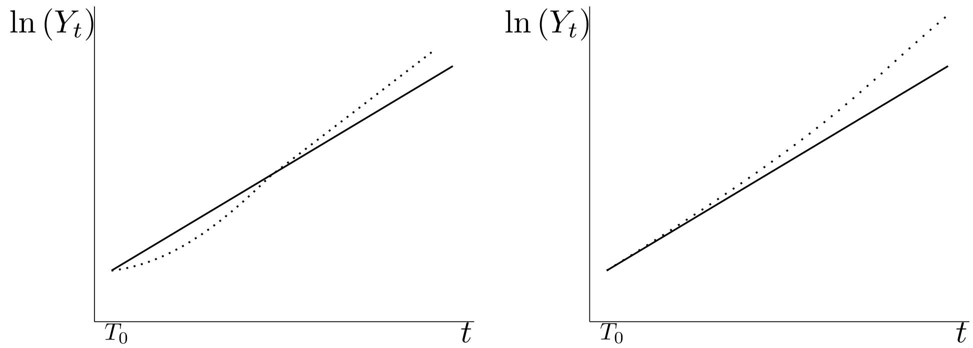
$$\frac{dN^\star}{ds_Z} < 0; \tag{35}$$

$$\frac{dg^\star}{ds_Z} > 0. \tag{36}$$

Proof: See the appendix.

Corollary 4 demonstrates the other trade-off, R&D subsidies decrease the steady-state number of firms, but increases the economy's growth rate. Specifically, the R&D subsidy increases the incentive to engage in R&D and consequently firms devote more resources to R&D. Because R&D is an endogenous fixed cost, as in Peretto (1996, 1998) and Sutton (2007), the increased R&D expenditure decreases net profit, which thereby reduces the incentive for entry and decreases the steady-state number of firms. Interestingly, the increase in the growth rate works through two channels: the direct effect of of the subsidy, and the increase in market size. The effects on income are depicted in figure (4).

Unlike the case of changes to the entry subsidy, output does not decline following the R&D subsidy. Intuitively, the increased R&D expenditure is offset by the reduction in the amount of resources devoted to horizontal innovation. Further, the economy's short-run growth response can be positive or negative; the R&D subsidy leads to the exit of firms which decreases the economy's growth rate, but the increase vertical innovation which increases the economy's growth rate. In figure (4a) the economy's short-run growth rate declines but the long-run growth rate is higher. In figure (4b) the short-run and long-run growth rates are both higher.



(a) Case 1 (b) Case 2

Figure 4: Dynamic response to R&D subsidy

Peretto (1998), Segerstrom (2000) and Chu, Furukawa, and Ji (2016) also analyze the effects of R&D subsidies in a Schumpeterian growth model with endogenous market structure. Peretto (1998) and Chu et al (2015) find a positive effect of R&D subsidies on economic growth. Segerstrom (2000) finds the R&D subsidies can have either positive or negative effects on economic growth, which is driven by the trade-off between quality improvement and variety expansion. This literature, however, assumes monopolistic competition and thus does not analyze the interaction between entry (and exit) and endogenous markups.

Due to endogenous markups in the oligopolistic competition, two distinct policies result in quite different *short-run* and *long-run* effects. Entry subsidies have a positive growth effect in the short run, but are unfavorable to the long-run growth. In contrast, R&D subsidies have a positive growth effect in the long run, but are unfavorable to growth in the short run. Endogenous markups thus give rise to a mis-adjustment for the two industrial policies: the short-term transition of the growth rate misadjusts from its long-term trend.

The tractability of our framework allows us to distinguish between short-run and long-run implications of policies which often oppose each other in our model. Because of their focus on steady states only, the existing literature fails to disentangle the two distinct time-frames. Our theoretical study, instead, has generated some interesting empirically testable hypotheses, and call for more empirical work.

5 Extension

In practice, we may observe a combination of industrial policies between entry and R&D subsidies. Thus, it is crucial to further understand the joined effect on the product variety and growth. To shed light on this issue, we engage in an additional experiment in this section.

Specifically, we impose that the subsidies take the following form:

$$s_N = s \ \ and \ \ s_Z = s(1+\theta),$$

where $-1 \leq \theta \leq 1$. This formulation implies that if $-1 \leq \theta \leq 0$, the change in the industry policy (captured by an increase in s) is more directed towards entry. By contrast, if $0 \leq \theta \leq 1$, the industry policy is more directed towards the in-house R&D subsidy instead of entry. This extension allows us to analyze the effect of simultaneous changes to entry and R&D subsidies in a systematic manner.[10]

We rewrite the steady-state number of firms (25) and the steady-state growth rate (28) as:

$$N^\star = \underset{N}{\text{argsolve}} \left[F(N,s) = \frac{\delta+\rho}{\psi} \right], \tag{37}$$

and

$$g^\star = \sigma\beta \left(\frac{\eta\sigma}{(1-(1+\theta)s) M(N^\star)} \right) \left(\frac{\beta}{1-\beta} \right) \left(\frac{L_Y^\star}{N^\star} \right) - \rho, \tag{38}$$

where

$$F(N,s) \equiv \left(\frac{\left[\frac{\rho(1-s)}{\psi} + (1-\tau)L\right] \beta (N + \sigma(\epsilon-1)(1-N))}{(1-s)(\epsilon(N-1)+1)} \right) - N \left(\frac{\eta\phi - \rho(1-(1+\theta)s)}{(1-s)\eta} \right). \tag{39}$$

Thus, we can re-derive the steady-state effect of s on the product variety and growth as follows:

$$\frac{dN^\star}{ds} = -\frac{\partial F}{\partial s} \Big/ \frac{\partial F}{\partial N} \ ,$$

$$\frac{dg^\star}{ds} = \frac{dg^\star}{dN^\star}\frac{dN^\star}{ds} + \frac{dg^\star}{dL_Y^\star}\frac{dL_Y^\star}{ds} + \frac{\partial g^\star}{\partial s}.$$

See the Appendix for more details.

If $\theta = -1$, as shown in the Appendix, the joint effect on product variety is unambiguously positive while the growth effect is unambiguously negative, which is a special case of Corollary 3. When the industrial policy is more directed towards entry ($-1 \leq \theta \leq 0$), a simultaneous increase in the entry and R&D subsidies is more likely to enhance the product variety (the number of firms) but decrease the growth rate (under Corollary 1). Specifically, when θ is smaller, the effect of the product variety (resp. growth) is positive (resp. negative), provided

[10] This extension was pointed out to us by an anonymous referee, to whom we are grateful.

that the following condition holds:

$$N < \frac{\left[\frac{\frac{\rho}{\psi}\beta(\epsilon-1)(1+\sigma)\eta}{(1+\theta)\rho}\right]^{\frac{1}{2}} - 1}{\epsilon} + 1.$$

This implies that a simultaneous increase in both the entry and R&D subsidies gives rise to an ambiguous effect on economic growth, crucially depending on the *status quo* of the product variety. If the number of firms N is substantially large at the *status quo*, the entry-induced effect of a further subsidy on entry becomes very limited due to the law of diminishing marginal returns. Under such a situation, a combination of industrial policies between entry and R&D subsidies will end up with a reduction in the number of firms because the effect of the R&D subsidy is dominating. Once the number of firms decreases, the growth rate is boosted by the industrial policy under Corollary 1. By contrast, if the number of firms N is relatively small at the *status quo*, the combined subsidy policy may (given that we only have the sufficient condition) favor entry and turn out to increase, rather than decrease, the number of firms. A larger number of firms dilutes the market of incumbent firms. Because the firm's market size decreases, firms engage in less R&D and the economy's growth rate declines.

6 Conclusion

We analyze the effects of changes in industrial policy (vertical and horizontal R&D subsidies) on competition and growth. In contrast to the existing literature on competition and innovation (Aghion and Howitt (1998), Bucci (2013), and Minniti (2010)) the degree of competition, and hence markups, is affected by the number of firms which is endogenous. We then study the effects of government policy through the endogenous markup channel. The tractability of our setup allows us to fully characterize the economy's dynamics, which to our knowledge, has not been studied in the literature. An entry subsidy can make an economy temporarily poorer, then temporarily richer, then permanently poorer, when compared to the counterfactual steady state. Similarly, an R&D subsidy can initially reduce the economy's growth rate, but lead to a higher long-run growth rate, due to the exit of firms. These results indicate that policy makers must take into consideration that the effects of industrial policy depends on the time frame in question. Moreover, our framework suggests the need for further, and very carefully done, empirical work. Specifically, an entry subsidy might have a positive effect on income immediately but have a negative effect on income in the long-run.

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Appendix

Proof of Lemma 1

Substituting (7) into (9), and combining with (8) yield the Current-value Hamiltonian for this optimization problem:

$$H_i = X_i \left(p_i - Z_i^{-\sigma} w\right) - w\phi - (1 - s_z)\, wL_{zi} + \varphi\eta Z L_{Zi},$$

where φ is the multiplier. The first order conditions include:

$$\frac{\partial H_i}{\partial p_i} = 0 \Longleftrightarrow p_i = wZ_i^{-\sigma} M(N); \tag{40}$$

$$\frac{\partial H_i}{\partial L_{Zi}} = 0 \Longleftrightarrow \frac{w(1 - s_z)}{\eta Z} = \varphi; \tag{41}$$

$$\frac{\partial H_i}{\partial Z_i} = \sigma X_i Z_i^{-\sigma-1} w = r\varphi - \dot{\varphi}. \tag{42}$$

Substituting (5), (6), (40), and (41) into (42) yields (14). ■

Proof of Lemma 2

Note that

$$\frac{\dot{C}}{C} = \frac{\dot{L}_y}{L_y} + \frac{\dot{w}}{w} = r - \rho$$

so

$$\frac{\dot{L}_y}{L_y} + \frac{\dot{w}}{w} + \rho = r$$

We know that

$$r_N = \frac{\psi N}{(1 - s_N)\, w}\pi + \frac{\dot{w}}{w} - \frac{\dot{N}}{N} - \delta.$$

Also

$$\frac{\dot{N}}{N} = \psi L_N - \delta$$

So

$$\frac{\dot{L}_y}{L_y} + \rho = \frac{\psi N}{(1 - s_N)\, w}\pi - (\psi L_N).$$

The household's budget constraint can be rewritten as

$$C + I = w + \pi$$

$$I = (1-\tau)\, w + \pi - Y$$

So

$$I = (1-s_N)\, wL_N$$

$$L_N = \frac{(1-\tau)\, w + N\pi - Y}{w\,(1-s_N)}$$

$$\frac{\dot{L_y}}{L_y} + \rho = \frac{\psi N}{(1-s_N)\, w}\pi - \left(\psi \frac{(1-\tau)\, w + N\pi - Y}{w\,(1-s_N)}\right).$$

$$\frac{\dot{L_y}}{L_y} + \rho = \psi \left(\frac{\frac{wL_Y}{1-\beta} - (1-\tau)\, w}{w\,(1-s_N)}\right).$$

So

$$\frac{\dot{L_y}}{L_y} = \psi \left(\frac{L_Y - (1-\beta)\,(1-\tau)}{(1-\beta)\,(1-s_N)}\right) - \rho.$$

So

$$L_Y = \frac{\rho\,(1-\beta)\,(1-s_N)}{\psi} + (1-\beta)\,(1-\tau)$$

Proof of Proposition 1

The proof proceeds as follows. First we derive $\frac{\dot{Z}}{Z}$. (22) indicates that $\frac{\dot{L_y}}{L_y} = 0$. (6) indicates that $\frac{\dot{Y}}{Y} = \frac{\dot{L_y}}{L_y} + \frac{\hat{w}}{w}$. Taking into account the above two equations, $C = Y$ and combining the Euler Equation in (3) and r_Z in (14) yields

$$\begin{aligned}
r_z &= \left(\frac{\beta}{1-\beta}\right)\left(\frac{L_y}{N}\right)\left(\frac{\eta\sigma\,(\epsilon-1)\,(N-1)}{(1-s_z)\,(N\epsilon-(\epsilon-1))}\right) + \frac{\dot{w}}{w} - \frac{\dot{Z}}{Z} \\
&= \left(\frac{\beta}{1-\beta}\right)\left(\frac{L_y^{\star}}{N}\right)\left(\frac{\eta\sigma\,(\epsilon-1)\,(N-1)}{(1-s_z)\,(N\epsilon-(\epsilon-1))}\right) + r - \rho - \frac{\dot{Z}}{Z} \\
\Longrightarrow \frac{\dot{Z}}{Z} &= \left(\frac{\beta}{1-\beta}\right)\left(\frac{L_y^{\star}}{N}\right)\left(\frac{\eta\sigma\,(\epsilon-1)\,(N-1)}{(1-s_z)\,(N\epsilon-(\epsilon-1))}\right) - \rho. \qquad (43)
\end{aligned}$$

Substituting $V = (1-s_N)\, w/\psi N$, (6), (5), (12), (13), (9), (22), $C = Y$, and (3), into (17) yields

$$r_N = \frac{\psi N}{1-s_N}\left(\left(\frac{\beta}{1-\beta}\right)\left(\frac{L_y}{N}\right)\left(\frac{1}{\epsilon-(\epsilon-1)\,S_i}\right)-\phi-(1-s_Z)L_{Zi}\right)+r-\rho-\frac{\dot{N}}{N}-\delta$$
$$\implies \frac{\dot{N}}{N} = \frac{\psi N}{1-s_N}\left(\left(\frac{\beta}{1-\beta}\right)\left(\frac{L_y}{N}\right)\left(\frac{1}{\epsilon-(\epsilon-1)\,S_i}\right)-\phi-(1-s_Z)L_{Zi}\right)-\rho-\delta$$

Substituting in equation (43) and (8) into the previous equation yields (24). ■

Proof of Proposition 2

A stable system requires $\frac{d\widehat{N}}{dN} < 0$; taking the derivative of $\frac{\dot{N}}{N}$ with respect to N, yields:

$$\frac{d\widehat{N}}{dN} = \frac{\psi}{1-s_N}\left[\left(\frac{\beta L_y^\star}{1-\beta}\right)\frac{(1-\sigma(\epsilon-1))(N\epsilon-(\epsilon-1))-\epsilon[N-\sigma(\epsilon-1)(N-1)]}{(N\epsilon-(\epsilon-1))^2}-\left(\phi-\frac{\rho(1-s_Z)}{\eta}\right)\right].$$

The first term inside the brackets is negative because

$$\begin{aligned}(1-\sigma(\epsilon-1))(N\epsilon-(\epsilon-1))-\epsilon[N-\sigma(\epsilon-1)(N-1)] &< 0\\ \iff -(\epsilon-1)+\sigma(\epsilon-1)^2-\epsilon\sigma(\epsilon-1) &< 0\\ \iff (\epsilon-1)[-1+\sigma\epsilon-\sigma-\epsilon\sigma] &< 0\\ \iff (\epsilon-1)[-1-\sigma] &< 0.\end{aligned}$$

Parameter restriction (27) is necessary because otherwise the system would explode when N becomes large enough.

The second part of the proof requires us to show that there only exists one positive root to $\frac{\dot{N}}{N} = 0$. There are two roots: $N_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$ *and* $N_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$. Given that $a < 0$, $c > 0$, it is easy to see $N_1 < 0 < N_2$. So $N^\star = \frac{-b-\sqrt{b^2-4ac}}{2a}$ is the only stable and positive root. Substituting $N^\star$ into (23) yields (28). ■

Proof of Corollary 1

Taking derivative of $g^\star$ with respect to $N^\star$ in (28) yields

$$\frac{\partial g^\star}{\partial N^\star} = \frac{\eta\sigma\left(\epsilon-1\right)}{\left(1-s_z\right)}\left(\frac{\beta L_Y^\star}{1-\beta}\right)\left(\frac{-\epsilon N^2+2\epsilon N-\left(\epsilon-1\right)}{\left[N(N^\star\epsilon-\left(\epsilon-1\right)\right]^2}\right)$$

$$\begin{cases} <0 & if\ N^\star > 1+\frac{1}{\epsilon}\ or N^\star < 1-\frac{1}{\epsilon}; \\ >0 & if\ 1-\frac{1}{\epsilon} < N^\star < 1+\frac{1}{\epsilon}. \end{cases}$$

Note that $N^\star > 1+\frac{1}{\epsilon}$, therefore $\frac{dg^\star}{dN^\star} < 0$. ■

Proof of Corollary 2

ϵ affects $g^\star$ through two channels: the markup M and affects $g^\star$ directly–it also affects $N^\star$ and hence $g^\star$ indirectly. The following equation shows the direct effect. $\frac{\partial g^\star}{\partial M} < 0$ by observation, and

$$\frac{\partial g^\star}{\partial M}\frac{\partial M}{\partial \epsilon} = (\frac{\partial g^\star}{\partial M})\frac{-N(N-1)}{\left[\left(\epsilon-1\right)\left(N-1\right)\right]^2} > 0.$$

Applying Implicit function theorem on (25) and some algebra yields

$$\frac{\partial \dot{N}/N}{\partial N}dN + \frac{\partial\left[\frac{N-\sigma(\epsilon-1)(N-1)}{\epsilon(N-1)+1}\right]}{\partial\epsilon}d\epsilon = 0$$

where

$$\frac{\partial \dot{N}/N}{\partial N} = \frac{\beta\psi L_y^\star}{1-s_N}\left[\frac{\left(1-\sigma\left(\epsilon-1\right)\right)\left(N\epsilon-\left(\epsilon-1\right)\right)-\epsilon\left(N-\sigma\left(\epsilon-1\right)\left(N-1\right)\right)}{\left(1-\beta\right)\left(N\epsilon-\left(\epsilon-1\right)\right)^2} - \psi\left(\frac{\eta\phi-\rho\left(1-s_Z\right)}{\beta\psi L_y^\star\eta}\right)\right]$$

Straightforward algebra yields

$$\frac{dN^\star}{d\epsilon} = \frac{-\frac{\partial\left[\frac{N-\sigma(\epsilon-1)(N-1)}{\epsilon(N-1)+1}\right]}{\partial\epsilon}}{\frac{\beta\psi L_y^\star}{1-s_N}\left[\frac{(1-\sigma(\epsilon-1))(N\epsilon-(\epsilon-1))-\epsilon(N-\sigma(\epsilon-1)(N-1))}{(1-\beta)(N\epsilon-(\epsilon-1))^2} - \psi\left(\frac{\eta\phi-\rho(1-s_Z)}{\beta\psi L_y^\star\eta}\right)\right]}.$$

Proposition 2 implies that the denominator is positive. It is straightforward to see that $\partial\left[\frac{N-\sigma(\epsilon-1)(N-1)}{\epsilon(N-1)+1}\right]/\partial < 0$. Therefore $\frac{\partial N}{\partial\epsilon} < 0$. Recall that $\frac{\partial g^\star}{\partial N^\star} < 0$ in Corollary 1, thus

$$\frac{\partial g^\star}{\partial\epsilon} = \frac{\partial g^\star}{\partial N^\star}\frac{\partial N^\star}{\partial\epsilon} < 0.$$

Finally, we see that ϵ affects $g^\star$ negatively in both direct and indirect channels. ■

Proof of Corollary 3

Applying implicit function theorem on (25) implies

$$\frac{\partial \dot{N}/N}{\partial N}dN + \frac{\beta\psi\left(1-\tau\right)L}{\left(1-s_N\right)^2}ds_N = 0,$$

where

$$\frac{\partial \dot{N}/N}{\partial N} = \frac{\beta\psi L_y^\star}{1-s_N}\left[\frac{\left(1-\sigma\left(\epsilon-1\right)\right)\left(N\epsilon-\left(\epsilon-1\right)\right)-\epsilon\left(N-\sigma\left(\epsilon-1\right)\left(N-1\right)\right)}{\left(1-\beta\right)\left(N\epsilon-\left(\epsilon-1\right)\right)^2}-\psi\left(\frac{\eta\phi-\rho\left(1-s_Z\right)}{\beta\psi L_y^\star\eta}\right)\right].$$

Therefore

$$\frac{dN}{ds_N} = -\frac{\frac{\beta\psi(1-\tau)L}{(1-s_N)^2}}{\frac{\beta\psi L_y^\star}{1-s_N}\left[\frac{(1-\sigma(\epsilon-1))(N\epsilon-(\epsilon-1))-\epsilon(N-\sigma(\epsilon-1)(N-1))}{(1-\beta)(N\epsilon-(\epsilon-1))^2}-\psi\left(\frac{\eta\phi-\rho(1-s_Z)}{\beta\psi L_y^\star\eta}\right)\right]}.$$

Proposition 2 implies that the denominator is negative. The numerator is positive. Therefore $dN/ds_N > 0$.

The second part of corollary 3 is proved as follows:

$$\frac{\partial g^\star}{\partial s_N} = \underbrace{\frac{\partial g^\star}{\partial L_y^\star}}_{>0}\underbrace{\frac{\partial L_y^\star}{\partial s_N}}_{<0} + \underbrace{\frac{\partial g^\star}{\partial N^\star}}_{<0}\underbrace{\frac{\partial N^\star}{\partial s_N}}_{>0} < 0.$$

■

Corollary 4

Applying implicit function theorem on (25) implies

$$\frac{\partial \dot{N}/N}{\partial N}dN - \frac{\psi N}{1-s_N}\frac{\rho}{\eta}ds_Z = 0$$

where

$$\frac{\partial \dot{N}/N}{\partial N} = \frac{\beta\psi L_y^\star}{1-s_N}\left[\frac{\left(1-\sigma\left(\epsilon-1\right)\right)\left(N\epsilon-\left(\epsilon-1\right)\right)-\epsilon\left(N-\sigma\left(\epsilon-1\right)\left(N-1\right)\right)}{\left(1-\beta\right)\left(N\epsilon-\left(\epsilon-1\right)\right)^2}-\psi\left(\frac{\eta\phi-\rho\left(1-s_Z\right)}{\beta\psi L_y^\star\eta}\right)\right].$$

Therefore

$$\frac{dN^\star}{ds_Z} = \frac{\frac{\psi N}{1-s_N}\frac{\rho}{\eta}}{\frac{\beta\psi L_y^\star}{1-s_N}\left[\frac{(1-\sigma(\epsilon-1))(N\epsilon-(\epsilon-1))-\epsilon(N-\sigma(\epsilon-1)(N-1))}{(1-\beta)(N\epsilon-(\epsilon-1))^2}-\psi\left(\frac{\eta\phi-\rho(1-s_Z)}{\beta\psi L_y^\star\eta}\right)\right]}. \tag{44}$$

Proposition 2 implies that the denominator is negative. The numerator is positive. Therefore, $\frac{dN^\star}{ds_Z} > 0$.

The second part of the proof follows from:

$$\begin{aligned}\frac{\partial g^\star}{\partial s_Z} &= \frac{\partial g^\star}{\partial s_Z} + \frac{\partial g^\star}{\partial N^\star}\frac{\partial N^\star}{\partial s_Z} \\ &= \left(\frac{\eta\sigma\left(\epsilon-1\right)\left(N^\star-1\right)}{(1-s_z)^2[N^\star\epsilon-(\epsilon-1)]}\right)\left(\frac{\beta}{1-\beta}\right)\left(\frac{L_Y^\star}{N^\star}\right) + \frac{\partial g^\star}{\partial N^\star}\frac{\partial N^\star}{\partial s_Z}.\end{aligned}$$

$\left(\frac{\eta\sigma(\epsilon-1)(N^\star-1)}{(1-s_z)^2[N^\star\epsilon-(\epsilon-1)]}\right)\left(\frac{\beta}{1-\beta}\right)\left(\frac{L_Y^\star}{N^\star}\right) > 0$; Corollary 1 together with (44) implies $\frac{\partial g^\star}{\partial N^\star}\frac{\partial N^\star}{\partial s_Z} > 0$. Thus, $\frac{\partial g^\star}{\partial s_Z} > 0$. ■

Extension

Applying the implicit function theorem to (39), the sign of F_2 (the partial derivative of $F\left(N, s\right)$ with respect to s) determines whether or not increases in s leads to an increase or decrease to the number of firms. When $\theta = -1$, $F_2 > 0$ requires that

$$\left(1-\tau\right)\frac{L}{N}\frac{\beta\left[N\left(1-\sigma\left(\epsilon-1\right)\right)+\sigma\left(\epsilon-1\right)\right]}{\left(\epsilon\left(N-1\right)+1\right)} - \left(\frac{\eta\phi-\rho}{\eta}\right) > 0.$$

In the steady-state, we have

$$N\left[\left(1-\tau\right)\frac{L}{N}\frac{\beta\left(N+\sigma\left(\epsilon-1\right)\left(1-N\right)\right)}{\left(\epsilon\left(N-1\right)+1\right)} - \left(\frac{\eta\phi-\rho}{\eta}\right)\right] = \frac{\delta}{\psi} > 0.$$

Therefore when $\theta = -1$, F_2 is positive.

In general, for F_2 to be positive, we must have

$$\frac{\left[\frac{\frac{\rho}{\psi}\beta(\epsilon-1)(1+\sigma)\eta}{(1+\theta)\rho}\right]^{\frac{1}{2}} - 1}{\epsilon} + 1 > N^\star$$

The proof is as follows.

First:

$$F_2 = \left(\frac{1}{1-s}\right)\left[\frac{\delta+\rho}{\psi} - \frac{\rho}{\psi}\frac{\beta\left(N^\star+\sigma\left(\epsilon-1\right)\left(1-N^\star\right)\right)}{\left(\epsilon\left(N^\star-1\right)+1\right)} - N^\star\frac{\left(1+\theta\right)\rho}{\eta}\right]$$

$$\frac{dF_2}{d\theta} = \frac{\partial F_2}{\partial N^\star}\frac{\partial N^\star}{\partial\theta} + \frac{\partial F_2}{\partial\theta}$$

Second:

$$\frac{\partial F_2}{\partial \theta} = -\frac{\rho}{\eta} N^{\star} < 0$$

$$\frac{\partial F_2}{\partial N} = \left(\frac{1}{1-s}\right)\left[-\frac{\rho}{\psi}\beta \frac{\left[\epsilon\left(N^{\star}-1\right)+1\right]\left[1-\sigma\left(\epsilon-1\right)\right]-\left[N^{\star}+\sigma\left(\epsilon-1\right)\left(1-N^{\star}\right)\right]\epsilon}{\left(\epsilon\left(N^{\star}-1\right)+1\right)^2} - \frac{\left(1+\theta\right)\rho}{\eta}\right]$$

$$\frac{\partial F_2}{\partial N^{\star}} = \left(\frac{1}{1-s}\right)\left[\frac{\rho}{\psi}\beta \frac{(\epsilon-1)(1+\sigma)}{\left(\epsilon\left(N^{\star}-1\right)+1\right)^2} - \frac{\left(1+\theta\right)\rho}{\eta}\right]$$

$$\frac{\partial N^{\star}}{\partial \theta} = \frac{2a\left[-\psi s\frac{\rho}{\eta}\left(\epsilon-1\right)-\frac{1}{2}\left(b(\theta)^2-4a(\theta)c\right)^{-\frac{1}{2}}\left(2b\psi s\frac{\rho}{\eta}\left(\epsilon-1\right)+4c\psi s\frac{\rho}{\eta}\epsilon\right)\right]+\left[-b(\theta)-\sqrt{b(\theta)^2-4a(\theta)c}\right]2\psi}{(2a)^2}$$

recall that $a < 0$ so $-b(\theta) - \sqrt{b(\theta)^2 - 4a(\theta)c} < 0$, therefore

$$\frac{\partial N^{\star}}{\partial \theta} < 0$$

$$\frac{\partial F_2}{\partial \theta} = \underbrace{\frac{\partial F_2}{\partial N}}_{?}\underbrace{\frac{\partial N^{\star}}{\partial \theta}}_{\text{¡0}} + \underbrace{\frac{\partial F_2}{\partial \theta}}_{\text{¡0}}$$

We must show the sign of $\frac{\partial F_2}{\partial \theta}$, it is positive when

$$\frac{\partial F_2}{\partial N^{\star}} = \left(\frac{1}{1-s}\right)\left[\frac{\rho}{\psi}\beta \frac{(\epsilon-1)(1+\sigma)}{\left(\epsilon\left(N^{\star}-1\right)+1\right)^2} - \frac{\left(1+\theta\right)\rho}{\eta}\right] > 0,$$

or

$$\frac{\left[\frac{\frac{\rho}{\psi}\beta(\epsilon-1)(1+\sigma)\eta}{(1+\theta)\rho}\right]^{\frac{1}{2}} - 1}{\epsilon} + 1 > N^{\star}.$$