Homework 5: Queueing model.

Question 1:

1.1. $\lambda = \frac{15 + 14 + 13}{3} = \frac{42}{3} = 14$ customers / 10 min

1.2. $\frac{1}{\lambda} = \frac{42}{10 \times 60} = 0.07$

1.3. $\rho = \frac{\lambda}{\mu} = \frac{14}{15} = 0.9333$

→ 93.33% of chances that a customer has to wait to receive service because the server are working for other clients. It also mean that only about 7% of chances that a customer can get service without waiting.

1.4. $L = \frac{\rho}{1-\rho} = \frac{0.9333}{1-0.9333} = 13.9925$

→ on average, there are 14 customers in the system (waiting in queue and receiving the service from the server).

$Lq = \frac{\rho^2}{1-\rho} = \frac{(0.9333)^2}{1-0.9333} = 13.0592$

→ on average, there are 13.0592 customers (about 13 customers) waiting in line.

1.5. $W = \frac{L}{\lambda} = \frac{13.9925}{14} = 0.9995$

→ on average, a customer has to spend 0.9995 of 10 minute to wait in line and get the service in the system.

good

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$Wq = \frac{Lq}{\lambda} = \frac{13.0592}{14} = 0.9328$

→ on average, a customer has to wait 0.9328 of 10 minutes (over 9 minutes) in the queue before receiving the service

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1.6. $P(n > 20) = \left(\frac{\lambda}{\mu}\right)^{20+1} = \left(\frac{14}{15}\right)^{21} = 0.9333^{21} = 0.2347$

→ There's 23% of chance that the number of customers in the system (waiting in line and being served) is greater than 20 customers

1.7. $P(W_q > 10)$

Because 1 period of time equal 10 minutes

→ $P(W_q > 10)_{min} \approx P(W_q > 1)$

$P(W_q > 1) = \rho \cdot e^{(\lambda - \mu)1} = (0.9333)(2.7183)^{(14-15)1} = (0.9333).(2.7183)^{-1}$

$= \frac{0.9333}{2.7183} = 0.3433.$

→ the probability that a customer has to stay in queue larger than 10 minutes is 34.33%.

great!

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1.8. $\rho = 0.9333$

$L = 7.4667$; $L_q = 6.533$

$W = 0.5333$; $W_q = 0.4667$

1.9. From Q1.3, Q1.4, Q1.5 and Q1.8 we can see that except for the value of ρ remain the same, the values of L, Lq, W and Wq are smaller with the smart servant. This means the customers can now spend less time in the system or waiting in line. The customers will love this servant.

On the other hand, ρ remains the same. This means the utilization factor or the traffic intensity for both the human employee and smart servant are the same. Therefore, the point of view of a manager, there's no difference between these two. Well...

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but good try

Question 2:

2.1. $\rho = 0.5$

$L = 2.1739$, $L_q = 0.1739$

$W = 1.087$; $W_q = 0.087$;

→ $L_q = 0.1739$ means average number of customer in line is 0.1739 customer which is less than 1 → satisfy guideline 1.

→ $P(n \leq 6) = 0.1304 + 0.2609 + 0.2609 + 0.1739 + 0.087 + 0.0435 + 0.0217$
$= 0.9782$

$P(n \leq 6) = 0.9782$ which is larger than 0.95 → guideline 2 satisfied.

→ $P(W \le 3) = 1 - P(W > 3) = 1 - 0.058 = 0.942$ ✓
→ smaller than 0.95 → guideline 3 is not satisfied. ✓

2.2. $\rho = 0.75$
$L = 4.5283$; $L_q = 1.5283$
$W = 1.5094$; $W_q = 0.5094$

→ $L_q = 1.5283 > 1$ → guideline 1 is not satisfied. ✓

→ $P(n \le 6) = 0.0377 + 0.1132 + 0.1698 + 0.1698 + 0.1274 + 0.0955 + 0.0716$
$= 0.7851$ ✓
→ $P(n \le 6) = 0.7851 < 0.95$ → guideline 2 not satisfied.

→ $P(W \le 3) = 1 - P(W > 3) = 1 - 0.1259 = 0.8741$ ✓ also smaller than 0.9
→ guideline not satisfied. ✓

2.3. After plugging in several number, we find the minimum number of servers that satisfy all guidelines is 8 servers.

→ With 8 servers, we have:
$L_q = 0.0078$ ✓
$P(n \le 6) = 0.9655$
$P(W > 3) = 0.95$

how? show me your reasoning.

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→ $\rho = 0.375$
$L = 3.0078$; $L_q = 0.0078$
$W = 1.0026$; $W_q = 0.0026$

Question 3.

For a single-server queueing system, a high utilization factor (ρ) can produce poor performance for the system. A higher ρ indicates that the server is working most of time serving clients, and if ρ is greater than 1 then the system has insufficient capacity and therefore cannot support customers properly.
If the server is human being, large ρ means that there would be no time for the server to have breaks or lunch. That would be harmful for the server. ✓

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Question 4

The common characteristic between Markov chain and queueing model regarding customers arrival is the one called "no memory property". No memory property says that customers arrivals are independent of one another or the number of arrivals in one period of time doesn't affect the number in the next period. Similarly, the Markov chain says that the conditional probability of any future event given any past event and present state, is independent of the past event and depend only upon the present state.