Homework (3 - 6 = 34

Public Policy Modeling

2 extra credits

Q. 1 (a) It is the only distribution of interarrival times that
(1.1) Fits having random arrivals.

(True)

The only distribution of interarrival times that fits having random arrivals is the exponential distribution. (section 11.1) 437 (1st para, last line)

(b). It has the lack-of memory property because it cannot remember when the next arrival will occur.

(False)

The probability of an arrival in the next minute is completely uninfluenced by when the last arrival occured is called the Lack of memory property.

1.2 (a) It generally provides an excellent approximation of the true service-time distribution.

(False)

Depending on the nature of queueing system, the exponential distribution can provide either a reasonable approximation of the true service-time distribution or a gross distortion of the true service-time distribution. (section 11.1) 438 -

(b) Its mean and variance are always equal.

(False)

In exponential distribution, standard deviation (σ) is equal to mean($\frac{1}{\lambda}$). The variance (σ^2) = $(\frac{1}{\lambda})^2$.

(c) It represents a rather extreme case regarding the amount of variability in the service times.

(False)

Other probability distribution also can be used to represent service time. For example, the Erlang distribution allows the amount of variability in the service times to fall somewhere between those for the exponential and degenerate distribution.

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1.3 The probability that exactly n clients ~~are~~ in the system is

$$P_n = (1-\rho)\rho^n \quad \text{for } n = 0, 1, 2, \dots$$

if 0 customers in the system $\Rightarrow$

$$P_0 = (1-\rho)\rho^0$$

$$P_0 = (1-\rho)\, 1 .$$

$$P_0 = 1-\rho$$

$$P_0 - 1 = 1 - 1 - \rho$$

$$P_0 - 1 = \rho$$

$\therefore$ The utilization factor ρ for the server in a single-server queueing system must equal $1 - P_0$.

Q.2. Assume : $\lambda = 8$ clients per day (8 hours)

$\mu = 10$ clients per day.

assumptions - $M/M/1/FCFS/\infty/\infty$

(2.1) Report the interarrival time and mean service time in hour.

interarrival time $= \dfrac{1}{\lambda} = \dfrac{1}{8} = 0.125$ day

interarrival time in hours $= 0.125$ day $\times$ 8 hours

$= 1$ hour.

$\therefore$ it means that a client arrives at the system every 1 hour.

mean service time $= \frac{1}{\mu} = \frac{1}{10} = 0.1$ day

mean service time in hours $= 0.1$ day $\times$ 8 hours

$= 0.8$ hours. ⋇

It means that on average 0.8 hours are needed for the system to service a client completely.

(2.2) Calculate ρ and then explain it meaning substantively.

utilization Factor $\rho = \frac{\lambda}{\mu} = \frac{8}{10} = 0.8$ ⋇

$\rho = 0.8$ means that 80 % of chance that the server (a tech rep) is working to serve a client (or).

80% of chance that a client has to wait to receive service since the server is working for other clients.

(2.3) Calculate L and L_q and then explain its meaning.

The expected number of clients in the system $= L$

$$L = \frac{\rho}{1-\rho} = \frac{0.8}{1-0.8} = \frac{0.8}{0.2} = 4. ⋇$$

$L = 4$ means that on average, there are 4 clients in the system who are waiting in the queue and receiving service.

The expected number of clients in the queue $= L_q$

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(0.8)^2}{1-0.8} = \frac{0.64}{0.2} = 3.2 ⋇$$

$L_q = 3.2$ means that on average, there are 3.2 clients in the queue to wait for the service.

(2.4) Calculate W and W_q and then explain its meaning.

Expected waiting time in the system = W

$$W = \frac{L}{\lambda} = \frac{4}{8} = 0.5 \text{ day} \times 8 \text{ hours}$$
$$= 4 \text{ hours.}$$

$W = 4$ means that a client has to spend on average 4 hours to wait and get service in the system.

Expected waiting time in the queue = W_q

$$W_q = \frac{\lambda W}{\mu} = \frac{8 \times 4}{10} = \frac{32}{10} = 3.2 \text{ hours}$$

$W_q = 3.2$ hours means that a client has to wait on average 3.2 hours in the queue before receiving service.

(2.5) Calculate P_7 and then explain its meaning.

$P_n = (1-\rho)\rho^n$, when exactly n clients are in the system.

$$P_7 = (1-\rho)\rho^7 = (1-0.8)\,0.8^7 = 0.2 \times 0.8^7 = 0.0419$$

4.19 % of chance that there are exactly 7 clients in the system.

(2.6) Calculate $P(n>5)$ and then explain its meaning.

$$P(n>5) = \left(\frac{\lambda}{\mu}\right)^{N+1} = \left(\frac{8}{10}\right)^{5+1} = \left(\frac{8}{10}\right)^6 = 0.2681.$$

$P(n>5) = 0.2681$ means that 26.81 percentage of chance that more than 5 clients will stay (to wait in a queue or to be served) in the system.

(8.7) Calculate P (w_q > 4 hour) and then explain its meaning.

$P(w_q > 4 \text{ hour}) = P(w_q > 0.5 \text{ days})$

$\rho e^{-\mu(1-\rho)T} = 0.8 \times 2.7182^{-10(1-0.8)0.5}$

$= 0.2943$ *

$P(w_q > 4 \text{ hours}) = 0.2943$ means that 29.43% of chance that a client will stay in the queue longer than 4 hours.

Q.3

3.1 assume: M/M/1/FCFS/∞/∞

W < 8 hours (the average waiting time in the system)

W < $\frac{8}{8}$ days (1 day = 8 hours)

W < 0.25 days.

μ = 10 (server can serve 10 clients per day)

λ = 5

⇒ Interarrival time in hour, $\frac{1}{\lambda} = \frac{1}{5} = 0.2$ day × 8 hours = 1.6 hours

∴ It means that a client arrives at the system every 1.6 hours.

⇒ $\rho = \lambda/\mu = 0.5$ *

50 % of chance that a client has to wait to receive service since the server is working for other client (50)

50 % of chance that the server is working to serve a client.

⇒ $L = L_q + \lambda/\mu = 1$ *

On average, there are 1 client in the system who is waiting in the queue and receiving service from the server.

⇒ $L_q = \frac{\rho^2}{1-\rho} = 0.5$ *

On average, there are 0.5 client in the queue to wait for the service.

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$\Rightarrow \quad W = L/\lambda = 0.2$ day

$= 0.2 \text{ day} \times 8 \text{ hours} = 1.6$ hours.

A client has to spend on average 1.6 hours to wait and get service in the system.

$\Rightarrow \quad W_q = L_q/\lambda = 0.1$ day

$= 0.1 \text{ day} \times 8 \text{ hours} = 0.8$ hours

A client has to wait on average 0.8 hours in the queue before receiving service.

attached excel sheet on pg (13)

(3.2) assume $M/G/1/FCFS/\infty/\infty$

$\sigma = 0.2$ (0.2 standard deviation of the service time)

mean service time $= \frac{1}{\mu} = 30$ minutes per customer

$\lambda = 8$ (on average 8 client per day (8 hours))

$\frac{1}{\mu} = 30$ minutes per customer

$\frac{1}{\mu} = \frac{30}{60} = 0.5$ hours per customer (0.0625 day per customer)

$\mu = 2$ (service can serve 2 clients per hour)

$\mu = 2 \times 8 \text{ hours} = 16$ (service can serve 16 clients per day)

$\rightarrow$ interarrival time in hour $= \frac{1}{\lambda} = \frac{1}{8} = 0.125 \text{ day} \times 8 \text{ hour} = 1$ hour

$\frac{1}{\lambda} = 1$ hour means that a client arrives at the system every 1 hour.

$\Rightarrow \quad \rho = 0.5$ (utilization factor)

50 % of chance that the server is working to serve a client

(or)

50 % of chance that a client has to wait to receive service since the server is working for other clients.

$\Rightarrow L = 3.31$

On average, there are 3.31 clients in the system who are waiting in the queue and receiving service from the server.

$\Rightarrow L_q = 2.81$

On average, there are 2.81 clients in the queue to wait for the service.

$\Rightarrow W = 0.41375$ day

$= 0.41375 \text{ day} \times 8 \text{ hours} = 3.31$ hours

A client has to spend on average 3.31 hours to wait and get service in the system.

$\Rightarrow W_q = 0.35125$ day

$= 0.35125 \text{ day} \times 8 \text{ hours} = 2.81$ hours

A client has to spend on average 2.81 hours in the queue before receiving service.

attached excel sheet on pg 15

(3.3) Assume M / D / 1 / FCFS / ∞ / ∞

$\lambda = 8$ (on average 8 clients per day (8 hours))

$\mu = 10$ (server can serve 10 clients per day)

$\Rightarrow$ interarrival time in hour, $\frac{1}{\lambda} = \frac{1}{8} = 0.125 \text{ day} \times 8 \text{ hours} = 1$ hour.

$\frac{1}{\lambda} = 1$ hour means that a client arrives at the system every 1 hour.

utilization Factor (ρ)

$\Rightarrow$ $\rho = 0.8$ *

80% of chance that the server is working to serve a client
(or)
20% of chance that a client has to wait to receive service since the server is working for other clients.

$\Rightarrow$ $L = 2.4$ *

On average, there are 2.4 clients in the system who are waiting in the queue and receiving service from the server.

$\Rightarrow$ $L_q = 1.6$ *

On average, there are 1.6 clients in the queue to wait for the service.

$\Rightarrow$ $W = 0.3$ day

$= 0.3 \times 8$ hours $= 2.4$ hours. *

A client has to spend on average 2.4 hours to wait and get service in the system.

$\Rightarrow$ $W_q = 0.2$ day

$= 0.2$ day $\times$ 8 hours $= 1.6$ hours.

A client has to wait on average 1.6 hours in the queue before receiving service.

attached excel sheet on pg 13

(3.4) Compare performance indicator obtained in Q 2.1 – Q 2.4, Q 3.1, Q 3.2 and Q 3.3.

Assume : Q 2.1 – Q 2.4 is system 1, Q.3.1 is system 2, Q 3.2 is system 3 and Q 3.3 is system 4

	Q.2.1 – Q.2.4 system (1)	Q-3.1 system (2)	Q-3.2 system (3)	Q-3.3 system (4)
Expected no of client in the system (per day) (λ)	8 client	5 client	8 client	8 client
mean service time ($\frac{1}{\mu}$)	0.8 hours	0.8 hour	0.5 hour	0.2 hour
utilization factor (ρ)	0.8	0.5	0.5	0.8
expected # of client in the system (L)	4 clients	1 client	3.31 client	2.4 client
expected # of client in the queue (Lq)	3.2 clients	0.5 client	2.81 client	1.6 client
expected waiting time in the system (W)	4 hours	1.6 hours	3.31 hours	2.4 hours
expected waiting time in the queue (Wq)	3.2 hours	0.8 hours	2.81 hours	1.6 hours

⇒ Firstly, compare system (1), (3) and (4).

- Even though they have the same no: of server (1 server) and same expected no of clients (8 clients per day), system (1) and (3) have longer waiting time than system (4).
- So I cancelled out system (1) and (3) from my consideration for choosing the best system

⇒ Secondly, I compare system (2) and (4).

Even though system (4) have longer waiting time than system (2), system (4) can service more 3 clients (8 clients – 5 clients) than system (2) and system (4) is more effectively used than system (2) ($\because \rho = 0.8$ (system 4) $> \rho = 0.5$ (system 2))

Make Sense

$\therefore$ I choose the system (4) as the best queueing system.

Q.4 Assume : M / M / s / FCFS / ∞ / ∞

- interarrival time is 20 minutes = $\frac{1}{\lambda}$ = 20 min.

change min to hour. $\Rightarrow \frac{20 \text{ min}}{60 \text{ min}} = 0.333$ hours

change hour to day. $= \frac{0.333 \text{ hours}}{8 \text{ hours}} = 0.04167$ days

$\therefore \frac{1}{\lambda} = 0.04167$ days

- $\mu = 10$, $\lambda = 24$

4.1. The minimal number of server that makes L smaller than 2.5 = 6

$P = 0.4$

40 % of chance that a client has to wait to receve service since the server is working for other client (or) 40 % of chance that the server is working to serve a client.

$L = 2.4266$

On average, there are 2.4 clients in the system who is waiting in the queue and receiving service from the server.

$L_q = 0.0266$

On average, there 0.0266 client in the queue to wait for the service.

$W = 0.1011$ day

$= 0.1011$ day $\times$ 8 hours $= 0.2022$ hours

A client has to spend on average 0.2022 hours to wait and get service in the system.

$W_q = 0.0011$ day

$= 0.0011$ day $\times$ 8 hours $= 0.0088$ hours.

A client has to wait on average 0.0088 hour in the queue before receiving service.

The expected no of clients in a system per day is 24 clients and the server provide service 10 clients per day. So to provide 24 clients per day, it is needed at least 3 servers. Now the system use 6 servers and utilization factor ρ is 0.4.

To be utilized efficitely the system, number of server should be reduced 6 server to 3 servers.

So what?

Q.4.2 Interarrival time = 20 mins

$$\frac{1}{\lambda} = \frac{20 \text{ mins}}{60 \text{ min}} = \frac{0.33 \text{ hours}}{8 \text{ hours}} = 0.04167 \text{ day.}$$

$\therefore \lambda = 24$, $\mu = 10$

Find minimal number of server that makes P(W_q > 30 minutes) smaller than 0.1 pecent (0.0001)

$$P(W_q > 30 \text{ minutes}) = P\left(W_q > \frac{30 \text{ minute}}{60 \text{ minute}} = 0.5 \text{ hours}\right)$$

$$= P\left(W_q > \frac{0.5 \text{ hour}}{8 \text{ hour}} = 0.0625 \text{ day}\right)$$

$\therefore$ The minimal number of server that makes P(W_q > 30 minutes = 0.0625 day) smaller than 0.1 percent is 9.

$\rho = 0.26667$

26.67 % of chance that a client has to wait to receive service since the server is waiting for other client (or)

26.67 % of chance that the server is working to serve a client.

L = 2.4003

On average, there are 2.4 clients in the system who is waiting in the queue and receiving service from the server.

$L_q = 0.0003$

On average, there are 0.0003 client (almost no client) in the queue to wait for the service.

$W = 0.1000$ days

$= 0.1000$ day $\times$ 2 hours $= 0.2$ hours.

A client has to spend on average 0.2 hours to wait and get service in the system.

$W_q = 1.36449\,E^{-05} = 0.0000136$ days

$= 0.0000136$ days $\times$ 2 hours $= 0.0001022$ hours)

A client has to wait on average 0.0001022 hour (almost doesn't need to wait) in the queue before receiving service.

+1

In this queueing system, the client doesn't need to wait in the queue and the client just need to wait for getting service. It will reduce the opportunity cost of the customer. However, the system use many servers (9 servers) and the utilization factor (ρ) is is very low ($\rho = 0.26667$). It means that the system not effectively used.

attached excel sheet on page 13

4.3. unit opportunity cost of a customer = \$ 350 per day (2 hours)

unit cost of a server(s) = \$ 100 per day.

obtain L when the number of server changes from 4 to 10.

expected service cost $= SC = C_s \times S$ (unit cost of server $\times$ # of server)

waiting cost (opportunity cost) = unit opportunity cost of a customer $\times$ expected no. of client in the system

$\therefore WC = C_w \times L$

Total social cost $= SC + WC$

Among number of server from 4 to 10, the optimal number of server is 5 because the total social cost for using 5 servers is the Lowest cost ($ 1376.6450 per day) among them.

attached Excel worksheet on pg 14

4.4. Draw a graph of expected service cost, waiting cost and total social cost.

attached Excel worksheet on pg 14

(13)

	A	B	C	D	E	F	G	H	I
1		**Queueing Model Template (Tech Rep)**							
2	Public Policy Modeling 2016 (DCC 5350)								
3	Question 3 and 4.2 of HW.5								
4	Win Thiri Myaing (1B 5077)								
5									
6		Mean arrival rate		λ =	5	8	8	λ =	24
7		Mean interarrival time		$1/\lambda$ =	0.2	0.125	0.125	$1/\lambda$ =	0.0416667
8		Mean service rate		μ =	10	16	10	μ =	10
9		Mean service time		$1/\mu$	0.1	0.0625	0.1	$1/\mu$	0.1
10									
11		Utilization factor		ρ =	0.5	0.5	0.8	ρ =	0.2666667
12		Number of servers		s =	1	1	1	s =	9
13		Standard deviation of the servic		σ =		0.2			
14									
15		# Clients in the queue		L_q =	0.5	2.81	1.6		0
16		# Client in the system		$L = L_q + \lambda/\mu$	1	3.31	2.4		2.4
17									
18		Waiting time in the queue		$W_q = L_q/\lambda$	0.1	0.35125	0.2		0
19		Waiting time in the system		$W = L/\lambda$	0.2	0.41375	0.3		0.1
20									
21		P(waiting time in the system > t)		Pr(W > t) =	0.0067379	0.000335463	0.1353353		0.2865048
22				When t =	1	1	1		0.125
23				Check	0.0067379	0.000335463	0.1353353		
24									
25		P(waiting time in the queue>t)		Prob(W_q > t) =	0.0410425	0.009157819	0.2943036		0.0161635
26				When t =	0.5	0.5	0.5		0.0625
27									
28		P(# clients in the system > N)		P(n > N) =	0.015625	0.015625	0.262144		
29				When N =	5	5	5		5
30				Check	1	1	1		1
31									

	A	B	C	D	E	F	G
1	**Public Policy Modeling 2016 (DCC5350)**						
2	**HW 5 Queueing Systems**						
3	**Sandy Suwarno (ID:1B5068)**						
4							
5	Data Table for Expected Total Cost for Alternative in Tech Representative						
6							
7		s	p	L	Cost of Service	Cost of Waiting	Total Cost
8		3	0.8	4.9887	$300	$1,746.05	$2,046
9		4	0.6	2.8306	$400	$990.71	$1,391
10		**5**	**0.48**	**2.5048**	**$500**	**$876.68**	**$1,377**
11		6	0.4	2.4266	$600	$849.31	$1,449
12		7	0.3428	2.4065	$700	$842.28	$1,542
13		8	0.3	2.4015	$800	$840.53	$1,641
14		9	0.2667	2.4003	$900	$840.11	$1,740
15		10	0.24	2.4	$1,000	$840.00	$1,840
16							
17	Unit Cost Server		$100				
18	Unit Cost Waiting		$350