100 − 0 = 100 ①

Public Policy Modeling - Homework 1. +1.5 - 1

Question 1:

$$-\frac{5}{2} \div -\frac{1}{3} + 4 = \frac{-5/2}{-1/3} + 4 = -\frac{5}{2} \cdot -\frac{3}{1} + 4$$

Apply basic arithmetic operation.

$$= \frac{(-5)\cdot(-3)}{2\cdot 1} + 4$$

$$= \frac{15}{2} + 4$$

$$= \frac{15}{2} + \frac{4}{1} = \frac{15\cdot 1 + 4\cdot 2}{2\cdot 1}$$ Apply basic arithmetic operation

$$= \frac{15+8}{2} = \boxed{\frac{23}{2}}$$

Question 2:

(2.1) $Y = A L^{\beta} K^{\alpha}$

$$Y \cdot \frac{1}{L^{\beta} K^{\alpha}} = A L^{\beta} K^{\alpha} \cdot \frac{1}{L^{\beta} K^{\alpha}}$$

Apply the cancelation law for multiplication

$$y = \frac{1}{L^{\beta} K^{\alpha}}$$

since $L^{\beta} K^{\alpha} \cdot \frac{1}{L^{\beta} K^{\alpha}} = 1.$

$$A = \frac{Y}{L^{\beta} K^{\alpha}}$$

(2.2) $\log Y = \log A + \beta \log L + \alpha \log K$

$\rightarrow \log Y - \log A - \beta \log L = \log A - \log A + \beta \log L - \beta \log L + \alpha \log K$

Cancelation law for addition : $y_1 = \log A$, $y_2 = \beta \log L$

$\rightarrow \alpha \log K = \log Y - \log A - \beta \log L$

$\rightarrow \alpha \cdot \log K \cdot \frac{1}{\log K} = (\log Y - \log A - \beta \log L) \cdot \frac{1}{\log K}$

Cancelation law for multiplication : $y = \frac{1}{\log K}$

$\rightarrow \alpha = \frac{\log Y}{\log K} - \frac{\log A}{\log K} - \frac{\beta \log L}{\log K}$

Distributive law

$= \log\left(\frac{Y}{K}\right) - \log\left(\frac{A}{K}\right) - \beta \log\left(\frac{L}{K}\right)$

Basic logarithm formulas.

$= \log\left(\frac{Y}{K} \div \frac{A}{K}\right) - \beta \log\left(\frac{L}{K}\right)$

$= \log\left(\frac{Y}{A}\right) - \beta \log\left(\frac{L}{K}\right)$

Question 3:

(3.1) $\frac{y}{3} + 1 = 2y - 2$

$3 \times \left(\frac{y}{3} + 1\right) = 3 \times (2y - 2)$

Cancelation law for multiplication : $y = 3$. and distributive law.

$y + 3 = 6y - 6$

$y + 3 - y = 6y - y - 6$

cancelation law for addition :

$5y - 6 = 3$

$5y - 6 + 6 = 3 + 6$

cancelation law for addition $y = 6$

$5y = 9$

$5 \times \frac{1}{5} \times y = 9 \times \frac{1}{5}$

cancelation law for multiplication : $y = \frac{1}{5}$.

$y = \frac{9}{5}$

not necessary.

(3.2) $\begin{cases} 4x - 2y = 10 & (1) \\ \frac{y}{2} + 1 = 3 - \frac{x}{2} & (2) \end{cases}$

- Rearrange and simplify ~~the~~ equation #2:

$\frac{y}{2} + 1 = 3 - \frac{x}{2}$

$\frac{y}{2} + \frac{x}{2} + 1 = 3 - \frac{x}{2} + \frac{x}{2}$ — Cancelation law for addition: $y = \frac{x}{2}$

$\frac{y}{2} + \frac{x}{2} + 1 = 3.$

$\frac{y}{2} + \frac{x}{2} + 1 - 1 = 3 - 1$ — Cancelation law for addition: $y = 1$

$\frac{y}{2} + \frac{x}{2} = 2$

$\frac{1}{2} \cdot y + \frac{1}{2} x = 2$ — apply distributive law

$\frac{1}{2}(x + y) = 2.$

$\frac{1}{2} \times 2 \times (x+y) = 2 \times 2.$ — Cancelation law for addition: $y = 2.$

$x + y = 4.$

→ the equations now become: $\begin{cases} 4x - 2y = 10. & (1) \\ x + y = 4 & (3) \end{cases}$

- Multiply both sides of equation (3), with 2, which is the smallest common multiple of y, then we have: (3) $(x + y) \times 2 = 4 \times 2$

$2x + 2y = 8$

→ the equations become: $\begin{cases} 4x - 2y = 10 & (1) \\ 2x + 2y = 8 & (4) \end{cases}$

- Add equation (1) to equation (4) to eliminate $2y$.

$(4x - 2y) + (2x + 2y) = 10 + 8$

$4x - 2y + 2x + 2y = 18$ — Commutative law applied.

$6x = 18.$

$6 \cdot \frac{1}{6} \cdot x = 18 \cdot \frac{1}{6}$ — Cancelation law for multiplication $y = \frac{1}{6}$

$x = 3.$ ✓

- Plug $x = 3$ into equation (1) to find y.

$4x - 2y = 10.$

$4(3) - 2y = 10.$ — Replace x with 3.

$12 - 2y = 10$

$12 - 12 - 2y = 10 - 12$ — cancelation law for addition: -12

$-2y = -2$

$-2 \cdot (-\frac{1}{2}) \cdot y = -2 \cdot (-\frac{1}{2})$ — cancelation law for multiplication $-\frac{1}{2}$.

$y = 1.$ ✓

→ therefore: $x = 3$; $y = 1.$

Question 4: $-2y + 10 \leq -4x.$ (1)

- Simplify equation (1) into the format $y \leq ax + b$ or $y \geqslant ax + b$.

$-2y + 10 \leq -4x.$

$-2y + 10 - 10 \leq -4x - 10$ — Cancelation law for addition applied: -10

$-2y \leq -4x - 10$

$-2 \cdot (-\frac{1}{2}) y \geqslant (-4x - 10)(-\frac{1}{2})$ — cancelation law for multiplication: $-\frac{1}{2}$. and change the sign of equation.

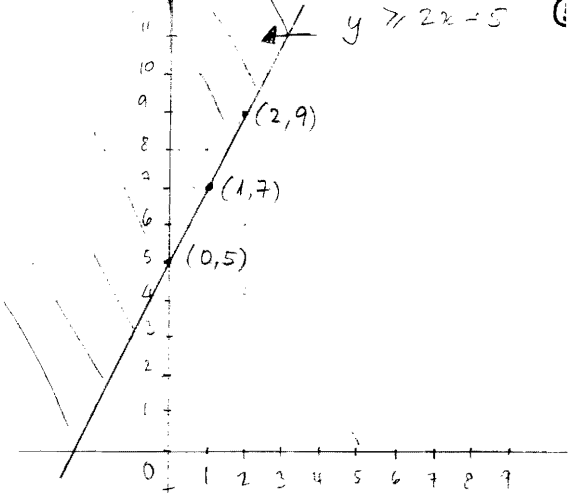
$y \geqslant 2x + 5$ (2) — apply distributive law

- Draw the plot of : $y = 2x + 5$.

$x = 0 \Rightarrow y = (2).0 + 5 = 5$.
$x = 1 \rightarrow y = (2).1 + 5 = 7$
$x = 2 \rightarrow y = (2).2 + 5 = 9$

- Plug in point (0,0) into the in equality
$y \geq 2x + 5$.
$0 \geq 2(0) + 5$
$0 \geq 5$ → doesn't make sense. ✓

→ the shaded area and the line : $y \geq 2x + 5$.



Question 5 : $A = \begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.10 & 0.70 & 0.20 \\ 0.05 & 0.35 & 0.60 \end{bmatrix}$; $B = \begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.30 & 0.55 & 0.15 \\ 0.05 & 0.35 & 0.60 \end{bmatrix}$

(5.1) $A' = \begin{bmatrix} 0.80 & 0.10 & 0.05 \\ 0.15 & 0.70 & 0.35 \\ 0.05 & 0.20 & 0.60 \end{bmatrix}$

(5.2) $A + B = \begin{bmatrix} 0.80 + 0.80 & 0.15 + 0.15 & 0.05 + 0.05 \\ 0.10 + 0.30 & 0.70 + 0.55 & 0.20 + 0.15 \\ 0.05 + 0.05 & 0.35 + 0.35 & 0.60 + 0.60 \end{bmatrix}$

$= \begin{bmatrix} 1.60 & 0.30 & 0.10 \\ 0.40 & 1.25 & 0.35 \\ 0.10 & 0.70 & 1.20 \end{bmatrix}$

(5.3) $A - B = \begin{bmatrix} 0.80 - 0.80 & 0.15 - 0.15 & 0.05 - 0.05 \\ 0.10 - 0.30 & 0.70 - 0.55 & 0.20 - 0.15 \\ 0.05 - 0.05 & 0.35 - 0.35 & 0.60 - 0.60 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ -0.20 & 0.15 & 0.05 \\ 0 & 0 & 0 \end{bmatrix}$ ✓

(5.4) $A \times B = \begin{bmatrix} 0.80 \times 0.80 + 0.15 \times 0.30 + 0.05 \times 0.05 & 0.80 \times 0.15 + 0.15 \times 0.55 + 0.05 \times 0.35 & 0.80 \times 0.05 + 0.15 \times 0.15 + 0.05 \times 0.6 \\ 0.10 \times 0.80 + 0.70 \times 0.30 + 0.20 \times 0.05 & 0.10 \times 0.15 + 0.70 \times 0.55 + 0.20 \times 0.35 & 0.10 \times 0.05 + 0.70 \times 0.15 + 0.20 \times 0.6 \\ 0.05 \times 0.80 + 0.35 \times 0.30 + 0.60 \times 0.05 & 0.05 \times 0.15 + 0.35 \times 0.55 + 0.60 \times 0.35 & 0.05 \times 0.05 + 0.35 \times 0.15 + 0.60 \times 0.6 \end{bmatrix}$

$= \begin{bmatrix} 0.6875 & 0.22 & 0.0925 \\ 0.30 & 0.47 & 0.23 \\ 0.175 & 0.41 & 0.415 \end{bmatrix}$ ✓

(5.5) → (5.8) : See attachement.

Question 6 :

Age group	Satisfied	Dissatisfied	Indifferent	
≤ 25	1,566	496	548	2610
26 - 29	6,636	3,017	2,413	12,066
30 - 39	36,406	19,274	15,705	71,385
	44,608	22,787	18,666	86,061.

Total respondent : 86,061

(6.1) • P(satisfied) = $\frac{44,608}{86,061}$ ✓ — # satisfied respondents / total # of respondents

= 0.5183.

• P(30_39) = $\frac{71,385}{86,061}$ ✓ — # respondent age 30_39 / # total respondents

= 0.8295

(6.2) • P(25 and under | satisfied)

= $\frac{P(25\text{and under} \,\&\, \text{satisfied})}{P(\text{satisfied})}$

P(25 and under & satisfied) = $\frac{1.566}{86,061}$ /

= 0.0182

P(satisfied) = $\frac{44,608}{86,061}$ = 0.5183. /

→ $\frac{P(25\text{and under} \,\&\, \text{satisfied})}{P(\text{satisfied})}$

= $\frac{0.0182}{0.5183}$ = 0.0351 ✓

• P(26_29 | satisfied)

= $\frac{P(26\_29 \,\&\, \text{satisfied})}{P(\text{satisfied})}$

P(26_29 & satisfied) = $\frac{6,636}{86,061}$ ✓

= 0.0771

→ $\frac{P(26\_29 \,\&\, \text{satisfied})}{P(\text{satisfied})}$ = $\frac{0.0771}{0.5183}$

= 0.1488 ✓

(6.3) P(satisfied | 30_39)

= $\frac{P(\text{satisfied and } 30\_39)}{P(30\_39)}$

P(satisfied and 30_39) = $\frac{36,406}{86,061}$

= 0.4230

→ P(satisfied | 30_39) = $\frac{0.4230}{0.8295}$

= 0.5099 ✓

(6.4) Age group is statistically independent of job satisfaction if: P(job satisfaction) = P(job satisfaction | group age)

P(satisfied) = P(satisfied | 30_39)

= $\frac{P(\text{satisfied} \,\&\, 30\_39)}{P(30\_39)}$

P(satisfied) = 0.5183.

P(satisfied | 30_39) = 0.5099

→ compare : P(satisfied) ≠ P(satisfied | 30_39) → ✓

age group <u>not statistically independent</u> of job satisfaction.

Public Policy Modeling - Homework 1

Question 5

$$A = \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.1 & 0.7 & 0.2 \\ 0.05 & 0.35 & 0.6 \end{bmatrix} \qquad B = \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.3 & 0.55 & 0.15 \\ 0.05 & 0.35 & 0.6 \end{bmatrix}$$

5.5 $|A| = 0.2725$ ✓ $\qquad |B| = 0.2$ ✓

5.6 $$AB = \begin{bmatrix} 0.6875 & 0.22 & 0.0925 \\ 0.3 & 0.47 & 0.23 \\ 0.175 & 0.41 & 0.415 \end{bmatrix} \checkmark \qquad BA = \begin{bmatrix} 0.6575 & 0.2425 & 0.1 \\ 0.3025 & 0.4825 & 0.215 \\ 0.105 & 0.4625 & 0.4325 \end{bmatrix} \checkmark$$

5.7 $$A^{-1} = \begin{bmatrix} 1.28440367 & -0.266055 & -0.0183486 \\ -0.183486239 & 1.752294 & -0.5688073 \\ 0 & -1 & 2 \end{bmatrix} \checkmark \qquad B^{-1} = \begin{bmatrix} 1.3875 & -0.3625 & -0.025 \\ -0.8625 & 2.3875 & -0.525 \\ 0.3875 & -1.3625 & 1.975 \end{bmatrix} \checkmark$$

5.8 $$BA - AB = \begin{bmatrix} -0.03 & 0.0225 & 0.0075 \\ 0.0025 & 0.0125 & -0.015 \\ -0.07 & 0.0525 & 0.0175 \end{bmatrix}$$

$$(BA-AB)^{-1} = \begin{bmatrix} -1.04811E+17 & 26.87747 & 4.4919E+16 \\ -1.04811E+17 & 44.26877 & 4.4919E+16 \\ -1.04811E+17 & -25.29644 & 4.4919E+16 \end{bmatrix} \checkmark$$