$100 - 1 = 99$ $+1$ extra credit

1

Public Policy Modeling
Homework 1 : Basic Math

Q1.

(1) $-\frac{2}{3} + 2 - \frac{7}{5}$

$= -\frac{2}{3}\times\frac{5}{5} + 2\times\frac{15}{15} - \frac{7}{5}\times\frac{3}{3}$

$= -\frac{10}{15} + \frac{30}{15} - \frac{21}{15}$

: Multiplied the question using the least common multiple of denominators whic the given question has (15 is the least common multiple of 3, 1 ($\because 2 = \frac{2}{1}$), and 5.)

$= \frac{30}{15} - \frac{10}{15} - \frac{21}{15}$: Commutative law applied to switch the order

$= \frac{-1}{15} = -\frac{1}{15}$: $-\frac{a}{b} = \frac{-a}{b}$ ($\because$ Basic arithmetic operations)

Therefore, $-\frac{2}{3} + 2 - \frac{7}{5} = -\frac{1}{15}$ ✓ //

(2) $-(-3)\times\left(\frac{2}{5}\right)^{-1} - 7$

$\left(\frac{2}{5}\right)^{-1} = 1 \div \frac{2}{5} = \frac{1}{\frac{2}{5}} = 1\times\frac{5}{2} = \frac{5}{2}$ ⋯ ① : $a^{-1} = 1 \div a = \frac{1}{a}$ ($\because$ Basic arithmetic operation) $a \neq 0$

$-(-3)\times\left(\frac{2}{5}\right)^{-1} - 7$

$= -(-3)\times\frac{5}{2} - 7$: Pluged in ① to the question

$= -\left(-3\times\frac{5}{2}\right) - 7$

$= -\left(-\frac{15}{2}\right) - 7$

$= \frac{15}{2} - 7$

$= \frac{15 - 14}{2\cdot 1}$: 2 is the least common multiple of 2 and 1 ($\because 7 = \frac{7}{1}$).

$= \frac{1}{2}$ ✓

Q2.

(1) $\frac{3}{2} - \frac{2x^2}{4x} = 2x$ $4x \Rightarrow x \neq 0$

$\frac{3}{2} - \frac{2x^2}{4x} + \frac{2x^2}{4x} = 2x + \frac{2x^2}{4x}$: Cancellation law for addition : $y = \frac{2x^2}{4x}$

$\frac{3}{2} = 2x + \frac{2x^2}{4x}$

$2x + \frac{2x^2}{4x} = \frac{3}{2}$: Definition of equation ($A = B \rightarrow B = A$)

$2x + \frac{2x^2}{4x} - \frac{3}{2} = \frac{3}{2} - \frac{3}{2}$: Cancellation law for addition : $y = -\frac{3}{2}$

$2x + \frac{2x^2}{4x} - \frac{3}{2} = 0$

$2x\cdot 4x + \frac{2x^2}{4x}\cdot 4x - \frac{3}{2}\cdot 4x = 0\cdot 4x$: Cancellation law for multiplication : $y = 4x$

$8x^2 + 2x^2 - 6x = 0$

$10x^2 - 6x = 0$

$2x \cdot 5x - 2x \cdot 3 = 0$: Distributive law applied

$2x(5x-3) = 0$

if $x = 0$, $2 \cdot \underline{\underline{0}} \cdot (5 \cdot 0 - 3) = 0$: If algebraic expressions composed of only multiplication include at least one zero, the solutions must be zero.

if $5x - 3 = 0$, $2 \cdot x \cdot \underline{0} = 0$

$5x - 3 + 3 = 0 + 3$: Cancellation law for addition : $y = 3$

$5x = 3$

$5 \times \frac{1}{5}x = 3 \times \frac{1}{5}$: Cancellation law for multiplication : $y = \frac{1}{5}$

$x = \frac{3}{5}$

in this case x cannot be 0

minor issue,

Therefore, $\cancel{x = 0}$, $\frac{3}{5}$ //

(2) $\begin{cases} 3x - 5y = -4 & \cdots ① \\ 5x = 2 - 7y & \cdots ② \end{cases}$

1st, the second equation should be rearranged by adding the positive of $7y$ to both side.

$5x = 2 - 7y$

$5x + 7y = 2 - 7y + 7y$: Cancellation law for addition applied

$5x + 7y = 2$

Now the equation system becomes :

$\begin{cases} 3x - 5y = -4 \\ 5x + 7y = 2 \end{cases}$

2nd, the both sides of each equation should be multiplied by the least common multiple of coefficients of y terms. 35 is the least common multiple of 5 and 7.

- $3x - 5y = -4$

  $(3x - 5y) \times 7 = -4 \times 7$: Multiply both sides by 7 to make $35y$

  $21x - 35y = -28$

- $5x + 7y = 2$

  $(5x + 7y) \times 5 = 2 \times 5$: Multiply both sides by 5 to make $35y$

  $25x + 35y = 10$

Now the equation system becomes :

$$\begin{cases} 21x - 35y = -28 \\ 25x + 35y = 10 \end{cases}$$

3rd, I add the first equation to the second equation side by side to eliminate $35y$

$(21x - 35y) + (25x + 35y) = -28 + 10$

$(21x + 25x - 35y + 35y) = -18$: Cummutative law applied

$46x = -18$

$46 \times \frac{1}{46} \times x = -18 \times \frac{1}{46}$: Cancellation law for multiplication applied

$x = -\frac{18}{46}$

$x = -\frac{18 \div 2}{46 \div 2} = -\frac{9}{23}$

4th, I solve one equation by replacing x with $-\frac{9}{23}$

$3x - 5y = -4$ The first equation

$3 \times (-\frac{9}{23}) - 5y = -4$: Replacement x with $-\frac{9}{23}$

$-\frac{27}{23} - 5y = -4$

$-\frac{27}{23} + \frac{27}{23} - 5y = -4 + \frac{27}{23}$: Cancellation law for addition applied $(+\frac{27}{23})$

$-5y = -\frac{4 \times 23}{1 \times 23} + \frac{27}{23}$

$-5y = -\frac{92}{23} + \frac{27}{23}$

$-5y = -\frac{65}{23}$

$-5 \times (-\frac{1}{5}) \times y = -\frac{65}{23} \times (-\frac{1}{5})$: Cancellation law for multiplication applied $(-\frac{1}{5})$

$y = \frac{13}{23}$

Therefore, $x = -\frac{9}{23}$ and $y = \frac{13}{23}$ // ✓

Q3.

(1) $F = m\frac{dv}{dt}$: The last formula

$\frac{dv}{dt} m = F$: Cummutative law applied

$\frac{dv}{dt} \div \frac{dv}{dt} \times m = F \div \frac{dv}{dt}$: the coefficient of m is $\frac{dv}{dt}$. Cancellation law for division applied

$\frac{dv}{dt} \times \frac{dt}{dv} \times m = F \times \frac{dt}{dv}$

$m = F\frac{dt}{dv}$ ✓

Ans. $\underline{m = F\frac{dt}{dv}}$ //

(2) $F = m\frac{dv}{dt}$: The last formula

$F \times dt = m\frac{dv}{dt} \times dt$: Cancellation law for multiplication applied (dt)

$F \cdot dt = m \cdot dv$: Cancellation law for multiplication applied ($\frac{1}{m}$) ($m \neq 0$)

$F \cdot dt \times \frac{1}{m} = m \times \frac{1}{m} \times dv$

$\frac{F}{m} dt = dv$

Ans. $dv = \frac{F}{m} dt$ ✓

Q4. $4x - 2y \leq 6$

$4x + (-4x) - 2y \leq 6 + (-4x)$: Cancellation law for addition ($-4x$)

$-2y \leq 6 - 4x$

$-2y \leq -4x + 6$: Cammutative law applied to switch the order

$-2y \times (-\frac{1}{2}) \geq (-4x + 6) \times (-\frac{1}{2})$: Cancellation law for multiplication ($-\frac{1}{2}$) The equality is switched.

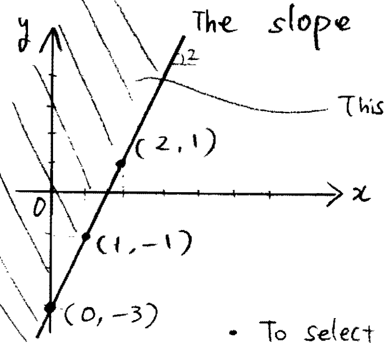
$y \geq -4x \times (-\frac{1}{2}) + 6 \times (-\frac{1}{2})$: Distributive law applied

$y \geq 2x - 3$

The linear equation $y = 2x - 3$

→ Y-interception of the line is -3

The slope of the line is 2



This shadow and on the line : $y \geq 2x - 3$

$x = 0 \Rightarrow y = 2 \cdot 0 - 3 = 0 - 3 = -3$

$x = 1 \Rightarrow y = 2 \cdot 1 - 3 = 2 - 3 = -1$

$x = 2 \Rightarrow y = 2 \cdot 2 - 3 = 4 - 3 = 1$

- To select a region, plug $(0, -4)$ in the inequality.

$y \geq 2x - 3$

$-4 \geq 2 \cdot 0 - 3$

$-4 \geq -3$ → Doesn't make sense.

Q4. (contd)

- To select a region, plug $(0,0)$ in the inequality.

$y \geq 2x - 3$

$0 \geq 2 \cdot 0 - 3$

$0 \geq -3$ ----> correct.

Therefore, these results indicate the region which contains the point $(0,0)$ and the left side of $y \geq 2x - 3$. ✓

please use single column !

Q5.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

(1) $A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$= \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix}$

$= \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$ //

$A - B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$= \begin{bmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{bmatrix}$

$= \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$ //

(2) $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$= \begin{bmatrix} 1\cdot5+2\cdot7 & 1\cdot6+2\cdot8 \\ 3\cdot5+4\cdot7 & 3\cdot6+4\cdot8 \end{bmatrix}$

$= \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}$

$= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$ //

$BA = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 5\cdot1+6\cdot3 & 5\cdot2+6\cdot4 \\ 7\cdot1+8\cdot3 & 7\cdot2+8\cdot4 \end{bmatrix}$

$= \begin{bmatrix} 5+18 & 10+24 \\ 7+24 & 14+32 \end{bmatrix}$

$= \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$ //

(3) $A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}'$

$= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ //

$B' = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}'$

$= \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$ //

(4) $|A| = 1\times4 - 2\times3$

$= 4 - 6$

$= -2$ //

$|B| = 5\times8 - 6\times7$

$= 40 - 42$

$= -2$ //

(5) $A^{-1} = \frac{1}{|A|} \operatorname{adj} A$

$|A| = 1\times4 - 2\times3$

$= 4 - 6$

$= -2$

$\operatorname{adj} A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Therefore,

$A^{-1} = \frac{1}{|A|} \times \operatorname{adj} A$

$= \frac{1}{-2} \times \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$= -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{2}\times4 & -\frac{1}{2}\times(-2) \\ -\frac{1}{2}\times(-3) & -\frac{1}{2}\times1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$ //

use single column

$$B^{-1} = \frac{1}{|B|} \text{adj } B$$

$$|B| = 5 \times 8 - 6 \times 7$$
$$= 40 - 42$$
$$= -2$$

$$\text{adj } B = \begin{bmatrix} 8 & -6 \\ -7 & 5 \end{bmatrix}$$

Therefore,

$$B^{-1} = \frac{1}{|B|} \text{adj } B$$
$$= \frac{1}{-2} \times \begin{bmatrix} 8 & -6 \\ -7 & 5 \end{bmatrix}$$ ∵ Replaced both $|B|$ & adj B

$$= -\frac{1}{2} \begin{bmatrix} 8 & -6 \\ -7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} \times 8 & -\frac{1}{2} \times (-6) \\ -\frac{1}{2} \times (-7) & -\frac{1}{2} \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 3 \\ 3.5 & -2.5 \end{bmatrix}$$ √ //

(6) $C = \begin{bmatrix} .1 & .2 & .3 & .4 \\ .4 & .3 & .2 & .1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$C' = \begin{bmatrix} .1 & .4 & 0 & 0 \\ .2 & .3 & 0 & 0 \\ .3 & .2 & 1 & 0 \\ .4 & .1 & 0 & 1 \end{bmatrix}$$ √

(7) ~ (9) were solved by Excel in another paper.

Q6.

	W	B	H
Voted	2735	(x) 366	233
Didn't vote	1403	257	345
total.	4138	623	578

(A total # of respondents : 5339)

(1) The number of people in Black ethnic group who voted (x) can be computated by using the given numbers.

· The # of white people :
$2735 + 1403 = 4138$

· The # of hispanic people :
$233 + 345 = 578$

· The # of black people :
$5339 - 4138 - 578 = 623$

Therefore,

$x = 623 - 257 = \underline{366}$

$$P(\text{Black}) = \frac{623}{5339}$$
$$= 0.116688\ldots$$
$$= 0.117 \;//$$

Public Policy Modeling
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Q5.7　get CC

$$C \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0.4 & 0.3 & 0.2 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0.4 & 0.3 & 0.2 & 0.1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Answer

$$\begin{bmatrix} 0.09 & 0.08 & 0.37 & 0.46 \\ 0.16 & 0.17 & 0.38 & 0.29 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✓

Q5.8　get determinant of C

Answer　−0.05 ✓

Q5.9　get C^{-1}

Answer

$$\begin{bmatrix} -6 & 4 & 1 & 2 \\ 8 & -2 & -2 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

✓

Q6.

(2) · $P(\text{Voted} \mid \text{White}) = \dfrac{P(\text{Voted} \cap \text{White})}{P(\text{White})}$: Conditional probability

$P(\text{Voted} \cap \text{White}) = \dfrac{2735}{5339}$: Joint probability of events "Voted" and "White". The denominator is the total # of respondents.

$= 0.5122\cdots$

$\fallingdotseq 0.512$: $0.5122\cdots$ rounded off to three decimal places is 0.512

$P(\text{White}) = \dfrac{4138}{5339}$: We've already gotten 4138, the # of white people in Q6 (1)

$= 0.7750\cdots$

$\fallingdotseq 0.775$: $0.7750\cdots$ rounded off to three decimal places is 0.775

Therefore,

$P(\text{Voted} \mid \text{White}) = \dfrac{P(\text{Voted} \cap \text{White})}{P(\text{White})}$

$= \dfrac{0.512}{0.775}$

$= 0.6606\cdots$

$\fallingdotseq 0.661$ // ✓

· $P(\text{Voted} \mid \text{Black}) = \dfrac{P(\text{Voted} \cap \text{Black})}{P(\text{Black})}$: Conditional probability

$P(\text{Voted} \cap \text{Black}) = \dfrac{366}{5339}$: Joint probability of events "Voted" and "Black".

$= 0.0685\cdots$

$= 0.069$: $0.0685\cdots$ rounded off three decimal places is 0.069.

$P(\text{Black}) = \dfrac{623}{5339}$: We've already gotten 623, the # of black people in Q6 (1)

$= 0.1166\cdots$

$\fallingdotseq 0.117$: $0.1166\cdots$ rounded off three decimal places is 0.117

Therefore,

$$P(\text{Voted} \mid \text{Black}) = \frac{P(\text{Voted} \cap \text{Black})}{P(\text{Black})}$$

$$= \frac{0.069}{0.117}$$

$$= 0.5897\cdots$$

$$\fallingdotseq 0.590$$

- $P(\text{Voted} \mid \text{Hispanic}) = \frac{P(\text{Voted} \cap \text{Hispanic})}{P(\text{Hispanic})}$: Conditional probability

$P(\text{Voted} \cap \text{Hispanic}) = \frac{233}{5339}$: Joint probability of events "Voted" and "Hispanic"

$$= 0.0436\cdots$$

$$\fallingdotseq 0.044$$

$P(\text{Hispanic}) = \frac{578}{5339}$: We've already gotten 578, the # of Hispanic people in Q6.(1)

$$= 0.1082\cdots$$

$$\fallingdotseq 0.108$$

Therefore,

$$P(\text{Voted} \mid \text{Hispanic}) = \frac{P(\text{Voted} \cap \text{Hispanic})}{P(\text{Hispanic})}$$

$$= \frac{0.044}{0.108}$$

$$= 0.4074\cdots$$

$$\fallingdotseq 0.407$$

(3) $P(\text{Black} \cap \text{Voted})$

$\text{Black} \cap \text{Voted} = \text{Voted} \cap \text{Black}$: Cammutative law to switch the order

Then, $P(\text{Black} \cap \text{Voted})$

$= P(\text{Voted} \cap \text{Black})$

$= \frac{366}{5339}$

$\fallingdotseq 0.069$

} : We've already gotten this result and process in Q6 (2)

Q6.

(4) $P(\text{Black} \mid \text{Voted}) = \dfrac{P(\text{Black} \cap \text{Voted})}{P(\text{Voted})}$: Conditional probability

$P(\text{Black} \cap \text{Voted}) = 0.069$ ∵ We've gotten this joint probability in Q6 (3)

$P(\text{Voted}) = \dfrac{2735 + 366 + 233}{5339}$

$= \dfrac{3334}{5339}$

$= 0.6244\cdots$

$\fallingdotseq 0.624$

Therefore,

$P(\text{Black} \mid \text{Voted}) = \dfrac{P(\text{Black} \cap \text{Voted})}{P(\text{Voted})}$

$= \dfrac{0.069}{0.624}$

$= 0.1105\cdots$

$\fallingdotseq 0.111$ // √

(5) If we can conclude that ethic group is statistically independent of voting, the event A (Voted) and the event B (ethnic group : Black) should be satisfied with $P(B \mid A) = P(B)$.

The result of Q6.1 is $P(\text{Black}) = 0.117$

The result of Q6.4 is $P(\text{Black} \mid \text{Voted}) = 0.111$

Now, we can compare these two numbers.

Then, $0.117 \neq 0.111$

Therefore, $P(\text{Black}) = 0.117 \neq P(\text{Black} \mid \text{Voted}) = 0.111$

Then, I conclude that ethic group (Black) is <u>not</u> statistically independent of voting