INTERNATIONAL UNIVERSITY OF JAPAN

Public Management and Policy Analysis Program
Graduate School of International Relations

DCC5350 (2 Credits)
Public Policy Modeling
Spring 2017

Homework 4: Markov Chain (120 points)

Instruction: Please write down your student ID and name at the top of your answer. PLEASE handwrite. Submit your answer to TA by 18:00 on May 26.

- You MUST always show your computation clearly. Otherwise, you may not get the full credits.
- Use at least four digits below the decimal point, if possible, in your probability calculation in order to avoid large rounding errors.
- You may not communicate (including written, verbal, gestural, any other communication) with others except for the instructor or TA to do this homework.

DO NOT ASK your classmates to show their work. DO NOT SHOW your work to other classmates. You may not share any electronic file with your classmate. Defend yourself by keeping your work in a safe place to avoid unintended cheating. Failure to comply with this rule will be considered academic dishonesty and misconduct. The penalties include sanctions up to and including expulsion from the university. I trust each of you implicitly but you should be aware of IUJ's policy on plagiarism and cheating.

1. Operating System Failure (45 points).

Suppose that a computer software vendor develops a brand-new operating system and claims that its average booting failure rate is below .1 percent. The operating system has only three states: Green (success), Blue, and Red (failure) in order. In order to test vendor's claim, a researcher installed the operating system into a standard desktop computer, turned on the computer every one hour, and check the booting state. A part of hourly data looks like "GGGBGGGBBRBBGBBGGGGGGGGBGG…" The experiment produces the following results. The number of transitions from Green to Green (#GG) is 950; #GB=45; #GR=5; #BG=170; #BB=20; #BR=10; #RG=0; #RB=10; #RR=40.

$$P = \begin{bmatrix} .95 & .045 & .005 \\ .85 & .10 & .05 \\ 0 & .20 & .80 \end{bmatrix}$$

1.1 **(10 points)** Explain how the transition probability matrix is constructed from the experiment data above.
1.2 **(5 points)** Draw the corresponding Markov transition diagram.
1.3 **(5 points)** Interpret P_{12} and P_{31} substantively.
1.4 **(5 points)** If the initial state is green or p_0 =[1 0 0], what is the probability that the operation system will be blue (similar to the blue screen of Microsoft Windows) after five

hours (t=5)? Provide relevant mathematical expression of this calculation and attach your Excel worksheet.

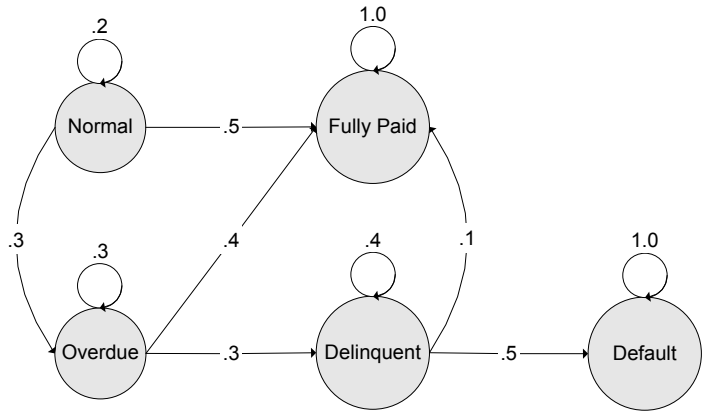
1.5 **(5 points)** Report 10-step and 20-step transition probability, $P^{(10)}$ and $P^{(20)}$. Attach your Excel worksheet. Handwrite $P^{(20)}$ and interpret the first column of $P^{(20)}$ substantively.

1.6 **(5 points)** Obtain three equations for long-run behavior. Rearrange the equations so that you can use $X=B^{-1}Y$.

1.7 **(5 points)** Calculate steady-state probabilities using Excel. Attach your Excel worksheet. Handwrite the result.

1.8 **(5 points)** Interpret the steady-state probability substantively. Is the operating system good enough (Is its failure rate less than .1 percent?)?

Question 2. Tax Collection Policy (50 points). The tax authority of a country wants to analyze tax collection system and then predict the amount of tax collected and default. It identifies five states: Normal, Fully Paid, Overdue, Delinquent, and Default. Their Markov transition diagram is given below. The time period is a quarter (three months).



2.1 **(5 points)** Construct the transition probability matrix. You must clearly determine the order of states.

2.2 **(5 points)** Report Q and R.

2.3 **(5 points)** Obtain $(I-Q)^{-1}$ using Excel. Attach your Excel Worksheet. Handwrite the result.

2.4 **(10 points)** Explain the first row of this fundamental matrix substantively.

2.5 **(5 points)** Obtain $(I-Q)^{-1}R$ using Excel. Attach your Excel Worksheet. Handwrite the result.

2.6 **(10 points)** Explain the second column of this absorption probability matrix substantively.

2.7 **(10 points)** Suppose the amount of tax imposed in Normal is on average USD 500 million, the amount of tax overdue is USD 200 million, and the amount of delinquent tax is USD 50 million. Report the predicted the amount of final tax collection and the amount of default. Show me your calculation clearly.

Question 3 New Tax Collection Policy (25 points) Suppose a new director of this tax authority changed a tax collection policy and introduced an electronic tax filing service called "EZ-tax." The total development and implementation cost was estimated to be USD 10 million. The tax authority found some changes in transition probabilities: P(Normal|Normal)=0, P(Overdue|Normal)=.1, P(Overdue|Overdue)=.1, P(Delinquent|Overdue)=.1, P(Delinquent|Delinquent)=.2, P(Fully Paid|Normal)=.9, P(Fully Paid|Overdue)=.8, P(Fully Paid|Delinquent)=.5, P(Default|Delinquent)=.3.

3.1 **(5 points)** Draw the new Markov transition diagram.
3.2 **(5 points)** Report new Q and R.
3.3 **(5 points)** Obtain $(I-Q)^{-1}$ and $(I-Q)^{-1}R$ using Excel. Attach your Excel Worksheet. Handwrite the results.
3.4 **(5 points)** Report the predicted the amount of final tax collection and the amount of default. Show me your calculation clearly.
3.5 **(5 points)** Do you think the new tax collection policy is successful? Why and why not?

Checklist.
1. Your answer sheet
2. Excel Worksheets for questions 1.4, 1.5, and 1.7
3. Excel Worksheets for questions 2.3 and 2.5
4. Excel Worksheets for question 3.3

End of homework assignment 4.