

Public Policy Modeling - Homework 2.

Question 1.

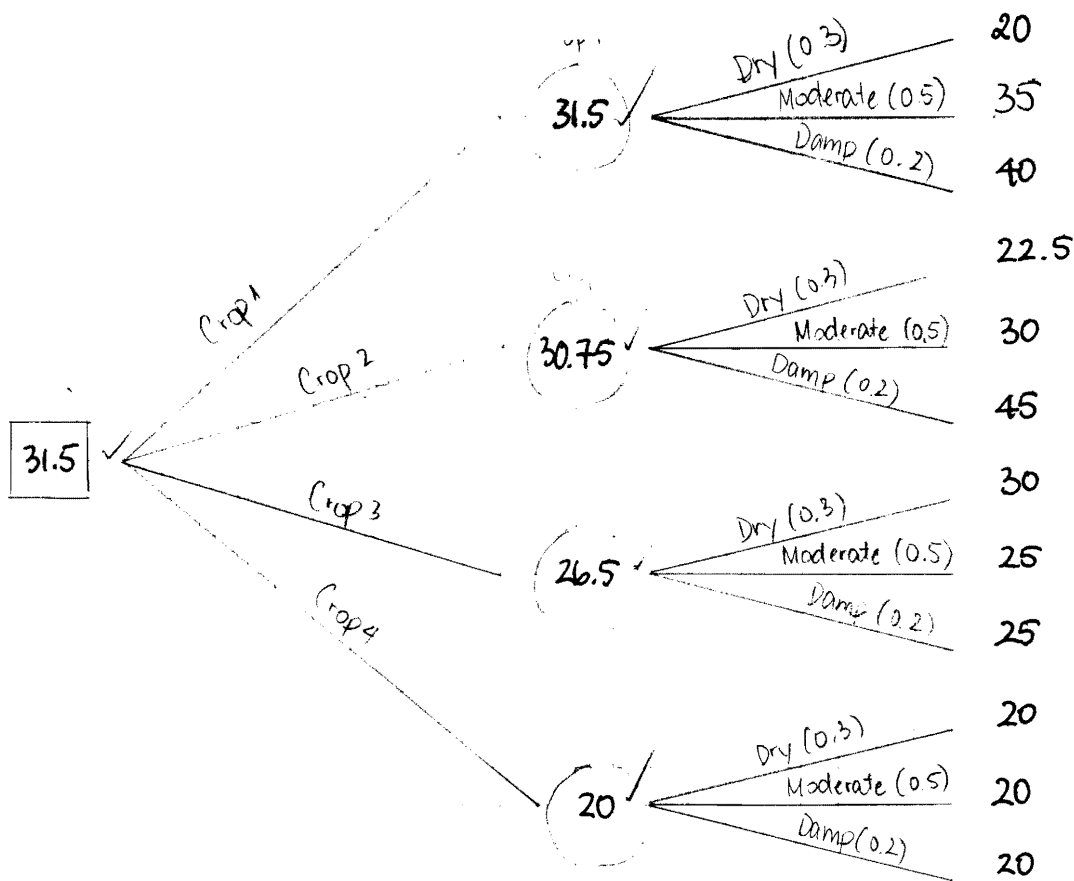
100 + 15 = 115 Great answers

(1.1) Construct payoff table

Alternatives	States of nature		
	Dry	Moderate	Damp
Crop 1	20	35	40
Crop 2	22.5	30	45
Crop 3	30	25	25
Crop 4	20	20	20
Prior probability	0.3	0.5	0.2

(unit in thousand dollars).

(1.2) Draw decision tree



45

Question 2

(2.1) Using Hurwicz criterion : $\alpha = 0.1 \rightarrow \alpha \times \text{Max}_i + (1-\alpha) \text{Min}_i$

Crop 1: $0.1 \times \text{Max}(20, 35, 40) + (1-0.1) \text{Min}(20, 35, 40) = 22$

Crop 2: $0.1 \times \text{Max}(22.5, 30, 45) + (1-0.1) \text{Min}(22.5, 30, 45) = 24.75$

Crop 3: $0.1 \times \text{Max}(30, 25, 25) + (1-0.1) \text{Min}(30, 25, 25) = 25.5$

Crop 4: $0.1 \times \text{Max}(20, 20, 20) + (1-0.1) \text{Min}(20, 20, 20) = 20$

→ Crop 3 has the largest weighted sum → Choose crop 3

(2.2) Using Maximin criterion

Crop 1: $\text{Min}(20, 35, 40) = 20$ ✓
 Crop 2: $\text{Min}(22.5, 30, 45) = 22.5$ ✓
 Crop 3: $\text{Min}(30, 25, 25) = 25$ ✓
 Crop 4: $\text{Min}(20, 20, 20) = 20$ ✓

→ Crop 3 has the 'largest' of the 'minimum payoff' → choose crop 3.

(2.3) Using Savage criterion

⊕ Regret table:

Regret (crop 1 | dry) = $\text{Max}(20, 22.5, 30, 20) - 20 = 10$ ✓
 Regret (crop 2 | dry) = $\text{Max}(20, 22.5, 30, 20) - 22.5 = 7.5$ ✓
 Regret (crop 3 | dry) = $\text{Max}(20, 22.5, 30, 20) - 30 = 0$ ✓
 Regret (crop 4 | dry) = $\text{Max}(20, 22.5, 30, 20) - 20 = 10$ ✓

Regret (crop 1 | moderate) = $\text{Max}(35, 30, 25, 20) - 35 = 0$ ✓
 Regret (crop 2 | moderate) = $\text{Max}(35, 30, 25, 20) - 30 = 5$ ✓
 Regret (crop 3 | moderate) = $\text{Max}(35, 30, 25, 20) - 25 = 10$ ✓
 Regret (crop 4 | moderate) = $\text{Max}(35, 30, 25, 20) - 20 = 15$ ✓

Regret (crop 1 | damp) = $\text{Max}(40, 45, 25, 20) - 40 = 5$ ✓
 Regret (crop 2 | damp) = $\text{Max}(40, 45, 25, 20) - 45 = 0$ ✓
 Regret (crop 3 | damp) = $\text{Max}(40, 45, 25, 20) - 25 = 20$ ✓
 Regret (crop 4 | damp) = $\text{Max}(40, 45, 25, 20) - 20 = 25$ ✓

I love these expressions!

→ Regret table

alternatives	Dry	Moderate	Damp
Crop 1	10	0	5
Crop 2	7.5	5	0
Crop 3	0	10	20
Crop 4	10	15	25

(thousand dollars)

⊕ Decide using savage criterion.

Maximum regret of Crop 1: $\text{Max}(10, 0, 5) = 10$ ✓
 Crop 2: $\text{Max}(7.5, 5, 0) = 7.5$ ✓
 Crop 3: $\text{Max}(0, 10, 20) = 20$ ✓
 Crop 4: $\text{Max}(10, 15, 25) = 25$ ✓

→ Crop 2 has the smallest maximum regret → choose crop 2.

Question 3:

(3.1) Decide using EMV.

EMV (Crop 1) = $0.3(20) + 0.5(35) + 0.2(40) = 31.5$ ✓
 EMV (Crop 2) = $0.3(22.5) + 0.5(30) + 0.2(45) = 30.75$ ✓
 EMV (Crop 3) = $0.3(30) + 0.5(25) + 0.2(25) = 26.5$ ✓
 EMV (Crop 4) = $0.3(20) + 0.5(20) + 0.2(20) = 20$ ✓

→ Crop 1 has the highest EMV → choose crop 1.

(3.2) Using EOL

$$\begin{aligned}
 EOL(\text{crop 1}) &= 0.3(10) + 0.5(0) + 0.2(5) = 4 \\
 EOL(\text{crop 2}) &= 0.3(7.5) + 0.5(5) + 0.2(0) = 4.75 \\
 EOL(\text{crop 3}) &= 0.3(0) + 0.5(10) + 0.2(20) = 9 \\
 EOL(\text{crop 4}) &= 0.3(10) + 0.5(15) + 0.2(25) = 15.5
 \end{aligned}$$

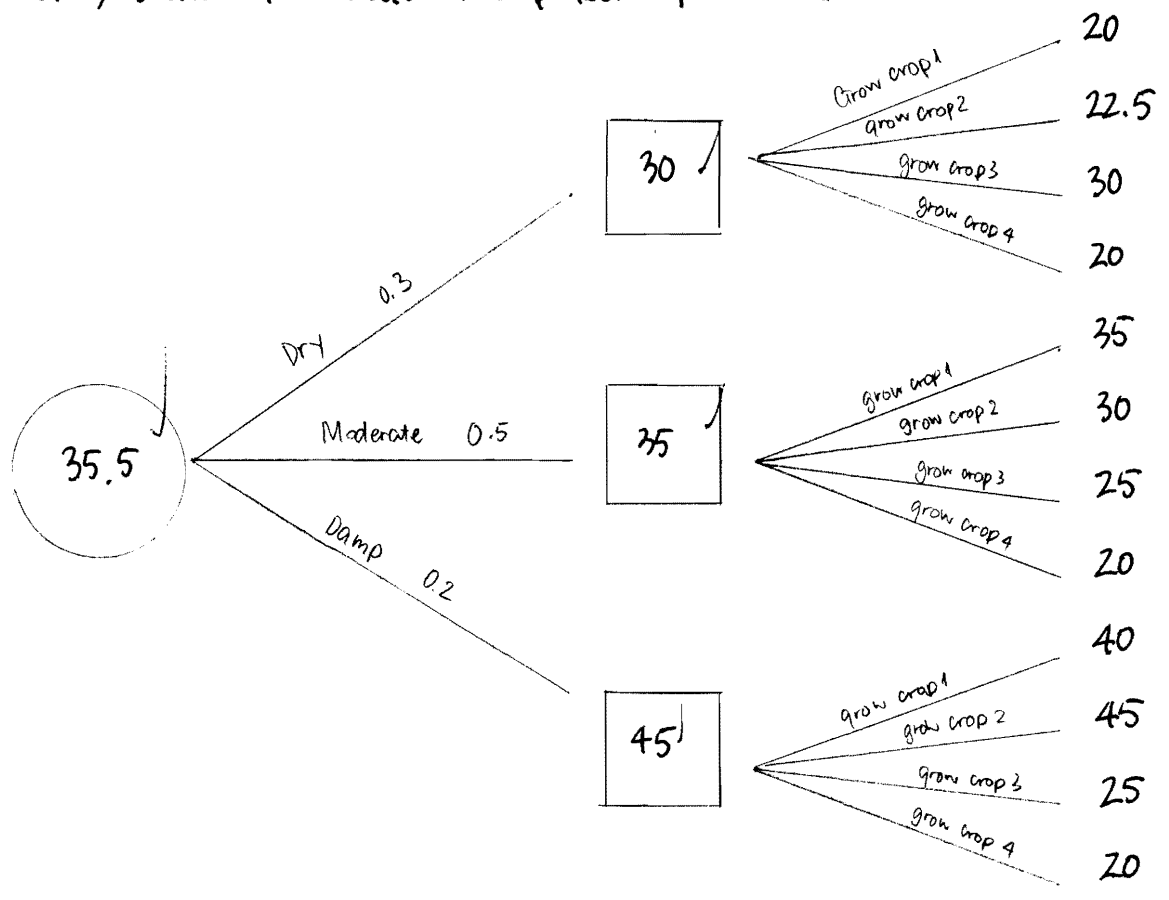
→ Crop 1 has the smallest EOL → choose crop 1.

Question 4

(4.1) Expected monetary value under /with perfect information.

$$\begin{aligned}
 EMV &= 0.3 \times \text{Max}(20, 22.5, 30, 20) + 0.5 \times \text{Max}(35, 30, 25, 20) + 0.2 \times \text{Max}(40, 45, 25, 20) \\
 &= 0.3(30) + 0.5(35) + 0.2(45) \\
 &= 35.5
 \end{aligned}$$

(4.2) Decision tree under the perfect information.



Question 5.

$$\begin{aligned}
 P(\text{green} | \text{dry}) &= 0.05 \\
 P(\text{green} | \text{moderate}) &= 0.7 \\
 P(\text{green} | \text{damp}) &= 1
 \end{aligned}$$

$$\begin{aligned}
 P(\text{red} | \text{dry}) &= 0.95 \\
 P(\text{red} | \text{moderate}) &= 0.3 \\
 P(\text{red} | \text{damp}) &= 0.
 \end{aligned}$$

(5.1) Six joint probabilities.

$$P(\text{green} \& \text{dry}) = P(\text{green} | \text{dry}) \cdot P(\text{dry}) = 0.05 \times 0.3 = 0.015$$

$$P(\text{green} \& \text{moderate}) = P(\text{green} | \text{moderate}) \cdot P(\text{moderate}) = 0.7 \times 0.5 = 0.35$$

$$P(\text{green} \& \text{damp}) = P(\text{green} | \text{damp}) \cdot P(\text{damp}) = 1 \times 0.2 = 0.2$$

$$P(\text{red} \& \text{dry}) = P(\text{red} | \text{dry}) \cdot P(\text{dry}) = 0.95 \times 0.3$$

$$P(\text{red} \& \text{moderate}) = P(\text{red} | \text{moderate}) \cdot P(\text{moderate}) = 0.3 \times 0.5 = 0.15$$

$$P(\text{red} \& \text{damp}) = 0$$

(4)

(5.2) Marginal probability.

$$P(\text{green}) = P(\text{green} \& \text{dry}) + P(\text{green} \& \text{moderate}) + P(\text{green} \& \text{damp}) \\ = 0.015 + 0.35 + 0.2 \\ = 0.565$$

$$P(\text{red}) = P(\text{red} \& \text{dry}) + P(\text{red} \& \text{moderate}) + P(\text{red} \& \text{damp}) \\ = 0.285 + 0.15 + 0 \\ = 0.435$$

(5.3) Six posterior probability.

$$P(\text{dry} | \text{green}) = \frac{P(\text{dry} \& \text{green})}{P(\text{green})} = \frac{0.015}{0.565} = 0.027$$

$$P(\text{dry} | \text{red}) = \frac{P(\text{dry} \& \text{red})}{P(\text{red})} = \frac{0.285}{0.435} = 0.655$$

$$P(\text{moderate} | \text{green}) = \frac{P(\text{moderate} \& \text{green})}{P(\text{green})} = \frac{0.35}{0.565} = 0.619$$

$$P(\text{moderate} | \text{red}) = \frac{P(\text{moderate} \& \text{red})}{P(\text{red})} = \frac{0.15}{0.435} = 0.345$$

$$P(\text{damp} | \text{green}) = \frac{P(\text{damp} \& \text{green})}{P(\text{green})} = \frac{0.2}{0.565} = 0.354$$

$$P(\text{damp} | \text{red}) = 0$$

(5.4) Expected monetary value of eight cases.

$$\text{EMV of crop 1 given green} = 0.027(20) + 0.619(35) + 0.354(40) = 36.37$$

$$\text{red} = 0.655(20) + 0.345(35) + 0(40) = 25.18$$

$$\text{EMV of crop 2 given green} = 0.027(22.5) + 0.619(30) + 0.354(45) = 35.11$$

$$\text{red} = 0.655(22.5) + 0.345(30) + 0(45) = 25.09$$

$$\text{EMV of crop 3 given green} = 0.027(30) + 0.619(25) + 0.354(25) = 25.14$$

$$\text{red} = 0.655(30) + 0.345(25) + 0(25) = 28.28$$

$$\text{EMV of crop 4 given green} = 0.027(20) + 0.619(20) + 0.354(20) = 20$$

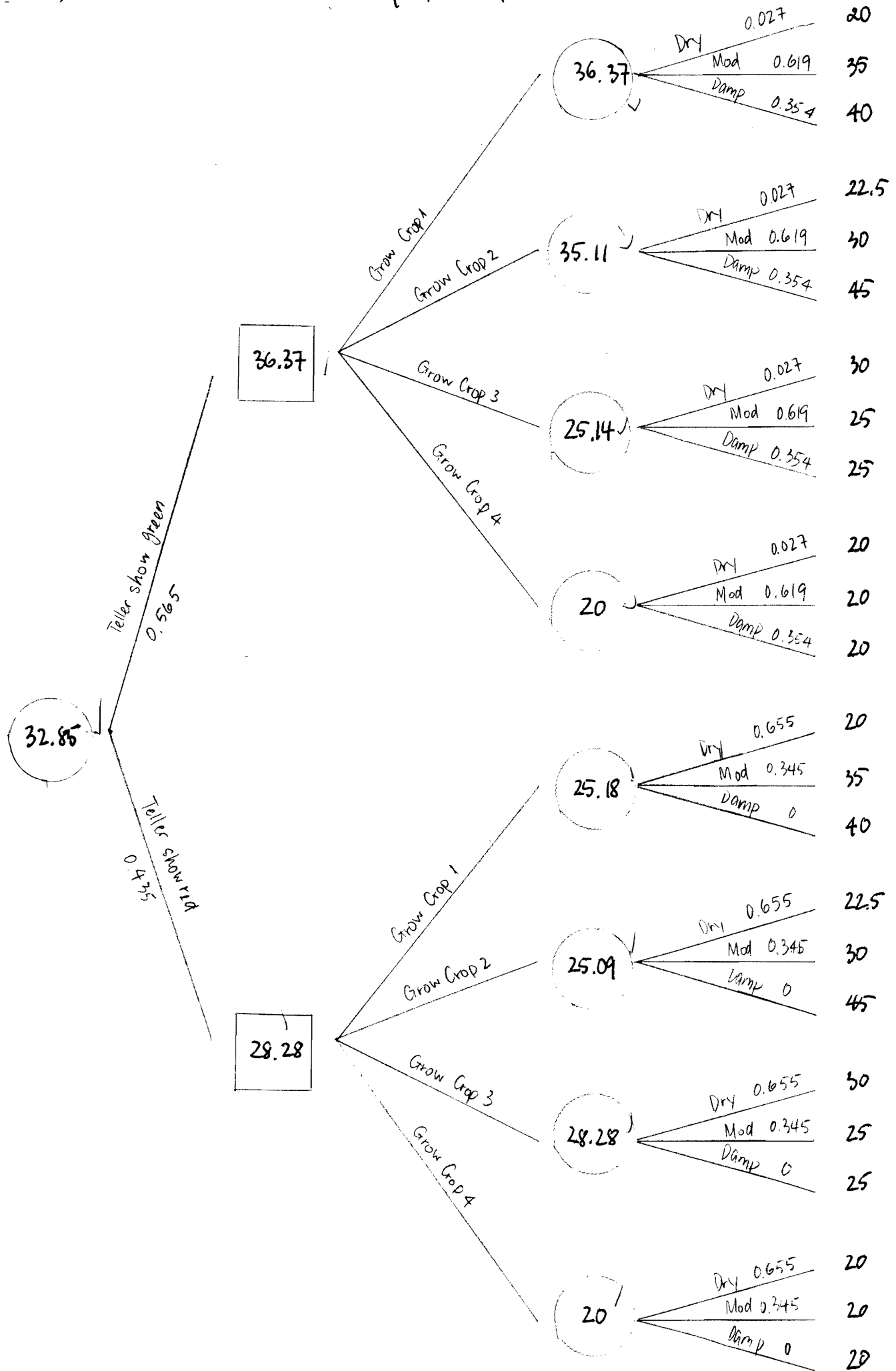
$$\text{red} = 0.655(20) + 0.345(20) + 0(20) = 20$$

(5.5) Expected value with this imperfect information.

$$\text{EMV} = 0.565 \times \max(36.37, 35.11, 25.14, 20) + 0.435 \times \max(25.18, 25.09, 28.28, 20) \\ = 0.565 \times (36.37) + 0.435 \times (28.28) \\ = 32.85$$

Great!

(5.6) Draw decision tree under imperfect information.



(5.7) $EV_{II} = \text{Expected value with imperfect information} - EMV$
 $= 32.85 - 31.5$
 $= 1.35 \checkmark$ But thousand!

→ EV_{II} is larger than the cost of weather Teller → buy weather Teller. ✓

Bonus question

$EVPI = \text{Expected value with perfect information} - EMV$
 $= 35.5 - 31.5$
 $= 4$ (thousand dollars)

EVPI set the maximum value for information which means that any information that cost more than EVPI is not worth buying.

If weather Teller's price is 2,000 USD → still smaller than 4,000 → I would probably advise Dwight to buy.

good reasoning.

+10