

Public Policy Modeling

Homework #2

$$100 - 5 = 95$$

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Question 1 Payoff table

Q.1.1 Solve the question a. Construct payoff table
(units in million dollars)

Decision Alternatives	States of Nature		
	Winning season	Losing season	Tired season
Hold campaign	3	-2	1
No campaign	0	0	0
Prior Probability	0.6	0.3	0.1

Question 2. Decision-making under uncertainty (Assume no probability)

Q.2.1. Decision with Savage criterion. Report the regret table.

$$\text{Regret (Hold campaign | winning)} = \text{Max}(3, 0) - 3 = 0$$

$$\text{Regret (No campaign | winning)} = \text{Max}(3, 0) - 0 = 3$$

$$\text{Regret (Hold campaign | Losing)} = \text{Max}(-2, 0) - (-2) = 2$$

$$\text{Regret (No campaign | Losing)} = \text{Max}(-2, 0) - 0 = 0$$

$$\text{Regret (Hold campaign | Tired)} = \text{Max}(1, 0) - 1 = 0$$

$$\text{Regret (No campaign | Tired)} = \text{Max}(1, 0) - 0 = 1$$

Regret table.

Alternatives	state of Nature		
	Winning season	Losing season	Tired season
Hold Campaign	0	2	0
No campaign	3	0	1

$$\text{Hold campaign: Max}(0, 2, 0) = 2$$

$$\text{No campaign: Max}(3, 0, 1) = 3$$

Therefore I choose Hold campaign because it gives smaller regret of 2 million dollars.

Question 2.2. Decision making with Laplace criterion

I will choose Laplace criterion, because I am not sure that even probability of having winning season is highest 0.6, \rightarrow Uncertainty also here is possibility to lose 2 million dollars. \rightarrow Maximin
This criterion used equal probability for each state of nature. 2 Not logical

Question 2.3. Make a decision with criterion in Q 2.2.

Laplace criterion, We assume that each season have same probability.

$$\begin{aligned}\text{Hold campaign: } & \frac{1}{3}(3) + \frac{1}{3}(-2) + \frac{1}{3}(1) = 1 + (-\frac{2}{3}) + \frac{1}{3} \\ & = \frac{3 \cdot (1) + (-2) + 1}{3} = \frac{3 - 2 + 1}{3} = \frac{2}{3} = 0.66\end{aligned}$$

$$\text{No campaign: } \frac{1}{3}(0) + \frac{1}{3}(0) + \frac{1}{3}(0) = 0 + 0 + 0 = 0$$

Therefore I will choose the alternative with maximum expected profit, which is 0.66 \rightarrow Hold campaign. \checkmark

Question 2.4. Decision making with Hurwicz criterion.

Assume that I have $\alpha = 0.3$ $1 - \alpha = 1 - 0.3 = 0.7$

As this criterion considers only maximum and minimum

$$\begin{aligned}\text{Hold campaign will give: } & 0.3 \cdot \text{Max}(3, -2, 1) + 0.7 \cdot \text{Min}(3, -2, 1) \\ & = 0.3 \cdot (3) + 0.7(-2) \\ & = 0.9 + (-1.4) \\ & = -0.5 \quad | \end{aligned}$$

$$\begin{aligned}\text{No campaign will give: } & 0.3 \cdot \text{Max}(0, 0, 0) + 0.7 \cdot \text{Min}(0, 0, 0) \\ & = 0.3 \cdot (0) + 0.7(0) \\ & = 0 + 0 \\ & = 0 \quad | \end{aligned}$$

Therefore I will choose No campaign, because it has higher value than negative (-0.5). \checkmark

Question 3. Decision making under risk.

Q. 3.1. Making decision using EMV

$$\text{Hold campaign: } 0.6 \cdot (3) + 0.3(-2) + 0.1(1) = 1.8 - 0.6 + 0.1 = 1.3 \quad \checkmark$$

$$\text{No campaign: } 0.6 \cdot (0) + 0.3(0) + 0.1(1) = 0 + 0 + 0 = 0 \quad \checkmark$$

Therefore I will choose Hold campaign $\text{Max}(1.3, 0) = 1.3$
it gives larger payoff of 1.3 million dollars \checkmark

Q. 3.2. Making a decision using EOL

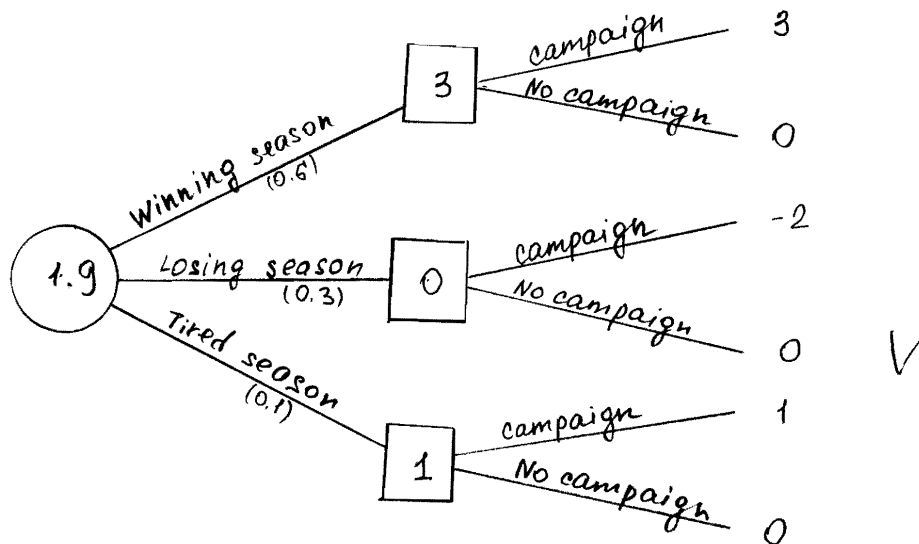
$$\text{Hold campaign: } 0.6 \cdot (0) + 0.3 \cdot (2) + 0.1 \cdot (0) = 0 + 0.6 + 0 = 0.6 \checkmark$$

$$\text{No campaign: } 0.6 \cdot (3) + 0.3 \cdot (0) + 0.1 \cdot (1) = 1.8 + 0 + 0.1 = 1.9 \checkmark$$

Therefore, I will choose hold campaign, because it has smaller expected opportunity loss of 0.6 million dollars.

Question 4. Expected value of perfect information

Q. 4.1. Decision tree under the perfect information.



Q. 4.2. Expected value of perfect information - calculation.

Expected value under the perfect information

$$\text{Max}(3, 0) \cdot (0.6) + \text{Max}(-2, 0) \cdot (0.3) + \text{Max}(1, 0) \cdot (0.1) = 1.8 + (-0.6) + 0.1 = 1.9$$

Expected value of perfect information

$$\text{EVPI} = 1.9 - 1.3 = 0.6 \checkmark$$

Question 5. Expected value of imperfect information.

William predictions as additional information:

$$P(\text{Up} | \text{Winning}) = 0.75$$

$$P(\text{Down} | \text{Winning}) = 1 - 0.75 = 0.25$$

$$P(\text{Down} | \text{Losing}) = 0.75$$

$$P(\text{Up} | \text{Losing}) = 1 - 0.75 = 0.25$$

$$P(\text{Up} | \text{Tired}) = 0.5$$

$$P(\text{Down} | \text{Tired}) = 0.5$$

Question 5.1 Identifying two decision variables

First decision variable for Athletic Department is to buy the additional information from William or not to buy this information.

Second decision variable for Athletic Department is to Hold campaign or not to hold campaign. ✓

Question 5.2. Identifying environment variables ^{which we can not control or decide}
In this decision situation environment variables are having winning season, having losing season or tired season. ✓

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Question 5.3. Calculate joint probabilities.

$$P(\text{Up \& winning}) = P(\text{Up} | \text{winning}) \times P(\text{winning}) = 0.75 \times 0.6 = 0.45 \quad /$$

$$P(\text{Down \& winning}) = P(\text{Down} | \text{winning}) \times P(\text{winning}) = 0.25 \times 0.6 = 0.15 \quad /$$

$$P(\text{Up \& losing}) = P(\text{Up} | \text{losing}) \times P(\text{losing}) = 0.25 \times 0.3 = 0.075 \quad /$$

$$P(\text{Down \& losing}) = P(\text{Down} | \text{losing}) \times P(\text{losing}) = 0.75 \times 0.3 = 0.225 \quad /$$

$$P(\text{Up \& Tired}) = P(\text{Up} | \text{Tired}) \times P(\text{Tired}) = 0.5 \times 0.1 = 0.05 \quad /$$

$$P(\text{Down \& Tired}) = P(\text{Down} | \text{Tired}) \times P(\text{Tired}) = 0.5 \times 0.1 = 0.05 \quad /$$

Question 5.4. Calculate marginal probabilities

$$P(\text{Up}) = P(\text{Up \& winning}) + P(\text{Up \& losing}) + P(\text{Up \& Tired}) = 0.45 + 0.075 + 0.05 = 0.575 \quad /$$

$$P(\text{Down}) = P(\text{Down \& winning}) + P(\text{Down \& losing}) + P(\text{Down \& Tired}) = 0.15 + 0.225 + 0.05 = 0.425 \quad /$$

Question 5.5. Calculate posterior probabilities

$$P(\text{winning} | \text{Up}) = \frac{P(\text{winning \& up})}{P(\text{up})} = \frac{0.45}{0.575} = 0.78 \quad /$$

$$P(\text{losing} | \text{up}) = \frac{P(\text{losing \& up})}{P(\text{up})} = \frac{0.075}{0.575} = 0.13 \quad /$$

$$P(\text{Tired} | \text{up}) = \frac{P(\text{Tired \& up})}{P(\text{up})} = \frac{0.05}{0.575} = 0.09 \quad /$$

$$P(\text{winning} | \text{Down}) = \frac{P(\text{winning} \& \text{Down})}{P(\text{Down})} = \frac{0.15}{0.425} = 0.35 \checkmark$$

$$P(\text{Losing} | \text{Down}) = \frac{P(\text{Losing} \& \text{Down})}{P(\text{Down})} = \frac{0.225}{0.425} = 0.53 \checkmark$$

$$P(\text{Tired} | \text{Down}) = \frac{P(\text{Tired} \& \text{Down})}{P(\text{Down})} = \frac{0.05}{0.425} = 0.12 \checkmark$$

Question 5.6 Expected value of four cases.

$$\begin{aligned} EV(\text{Hold campaign} | \text{up}) &= P(\text{winning} | \text{up}) \cdot (3) + P(\text{Losing} | \text{up}) \cdot (-2) + P(\text{Tired} | \text{up}) \cdot (1) = \\ &= (0.78) \cdot (3) + (0.13) \cdot (-2) + (0.09) \cdot (1) = 2.174 \checkmark \end{aligned}$$

$$\begin{aligned} EV(\text{No campaign} | \text{up}) &= P(\text{winning} | \text{up}) \cdot (0) + P(\text{Losing} | \text{up}) \cdot (0) + P(\text{Tired} | \text{up}) \cdot (0) = \\ &= (0.78) \cdot (0) + (0.13) \cdot (0) + (0.09) \cdot (0) = 0 \end{aligned}$$

$$\begin{aligned} EV(\text{Hold campaign} | \text{Down}) &= P(\text{winning} | \text{Down}) \cdot (3) + P(\text{Losing} | \text{Down}) \cdot (-2) + P(\text{Tired} | \text{Down}) \cdot (1) = \\ &= (0.35) \cdot (3) + (0.53) \cdot (-2) + (0.12) \cdot (1) = 0.117 \checkmark \end{aligned}$$

$$\begin{aligned} EV(\text{No campaign} | \text{Down}) &= P(\text{winning} | \text{Down}) \cdot (0) + P(\text{Losing} | \text{Down}) \cdot (0) + P(\text{Tired} | \text{Down}) \cdot (0) = \\ &= (0.35) \cdot (0) + (0.53) \cdot (0) + (0.12) \cdot (0) = 0 \end{aligned}$$

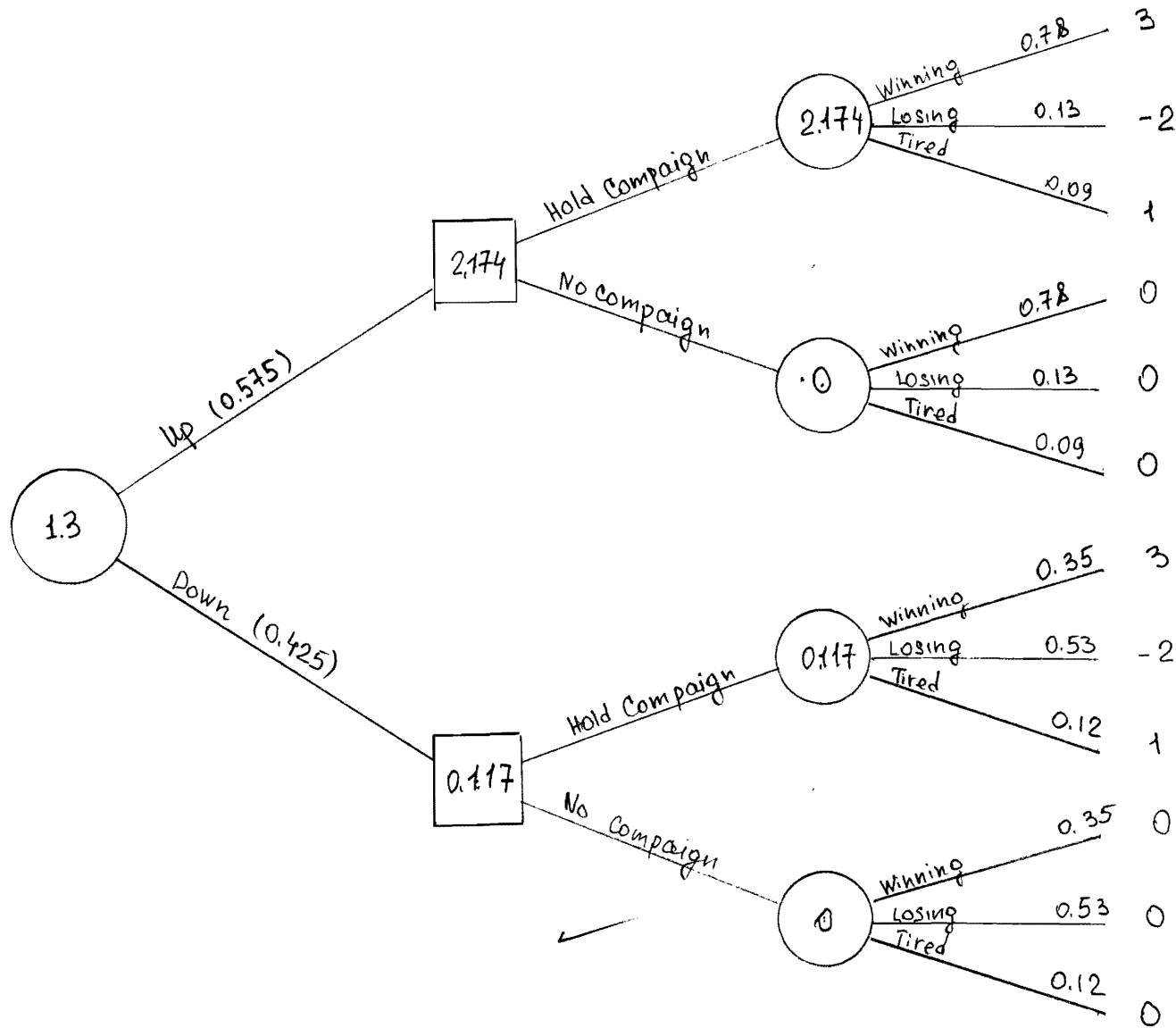
Question 5.7 Calculate this value with this imperfect information

$$\begin{aligned} EV(\text{with imperfect information}) &= [P(\text{up}) \cdot \text{Max}(2.174, 0)] + [P(\text{Down}) \cdot \text{Max}(0.117, 0)] \\ &= 0.575 \cdot 2.174 + 0.425 \cdot 0.117 = 1.3 \checkmark \end{aligned}$$

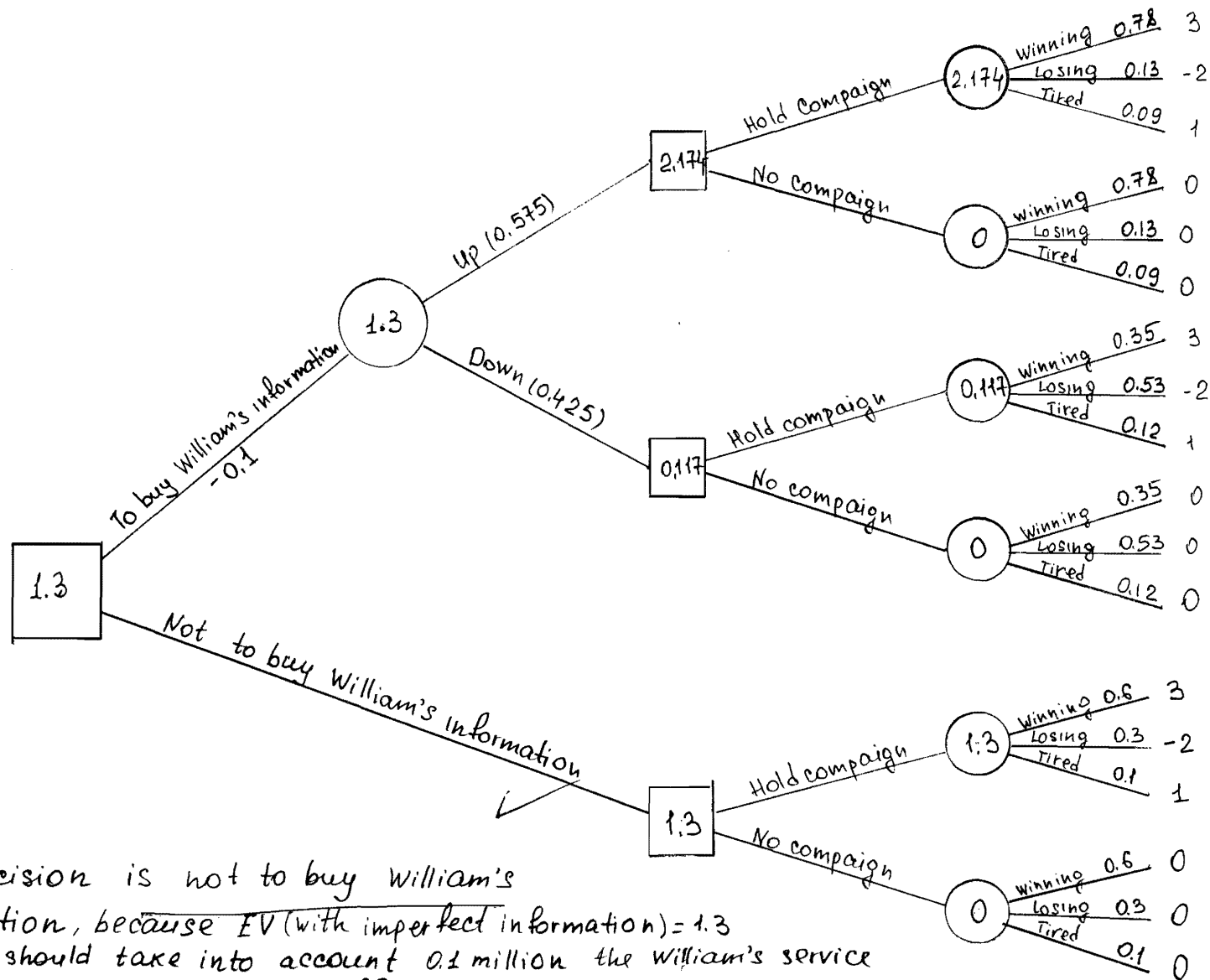
Question 5.8 Calculate expected value of sample information.

$$\begin{aligned} EVSI &= EV(\text{with imperfect information}) - EMV(\text{with no information}) \\ &= 1.3 - 1.3 = 0 \checkmark \end{aligned}$$

Question 5.9 Decision tree under the imperfect information



5.10. Decision tree with buying William's hunch incorporated.



Last decision is not to buy William's information, because $EV(\text{with imperfect information}) = 1.3$ also we should take into account 0.1 million the William's service it will give us 1.2 million payoff. Therefore Not to buy William's information gives higher payoff of 1.3 million dollars.