$150 - 2x = 126$

+ Tool

Question 1

Objective function , Maximize $P = 3x_1 + 2x_2$

First constraint , $x_1 + x_2 \leq 8$ (resource 1)

second constraint , $2x_1 + x_2 \leq 10$ (resource 2)

Max: $P = 3x_1 + 2x_2$

subject to

$x_1 + x_2 \leq 8$ (resource 1)

$2x_1 + x_2 \leq 10$ (resource 2)

and

$x_1 \geq 0$, $x_2 \geq 0$

to find the feasible region ,

for first constraint , $x_1 + x_2 \leq 8$

$x_1 + (-x_1) + x_2 \leq 8 + (-x_1)$,

$x_2 \leq 8 - x_1$

to find the Y intersection ,

when $x_1 = 0$, $x_2 = 8 - 0 = 8$

therefore , Y intersection = $(0, 8)$.

to find the x intersection,

when $x_2 = 0$, $0 = 8 - x_1$

$8 - x_1 = 0$

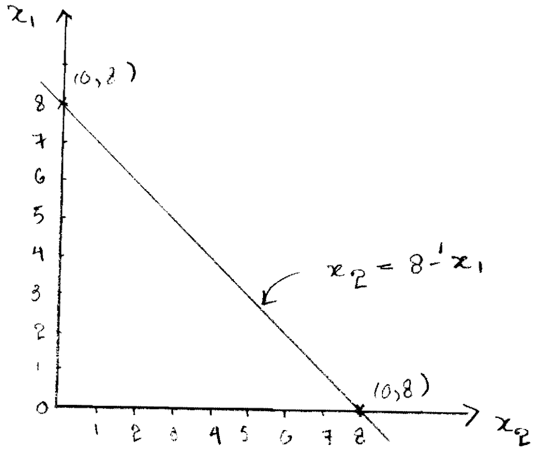
$8 + (-8) - x_1 = 0 + (-8)$

$-x_1 = -8$

$-x_1 \times (-1) = -8 \times (-1)$

$x_1 = 8$

therefore , x intersection = $(8, 0)$



for second constraint , $2x_1 + x_2 \leqslant 10$

$$2x_1 + (-2x_1) + x_2 \leqslant 10 + (-2x_1)$$

$$x_2 \leqslant 10 - 2x_1$$

to find the Y intersection,

when $x_1 = 0$, $x_2 = 10 - 2 \times 0 = 10$

therefore , Y intersection $= (0, 10)$

to find the x intersection,

when $x_2 = 0$, $0 = 10 - 2x_1$

$$10 - 2x_1 = 0$$

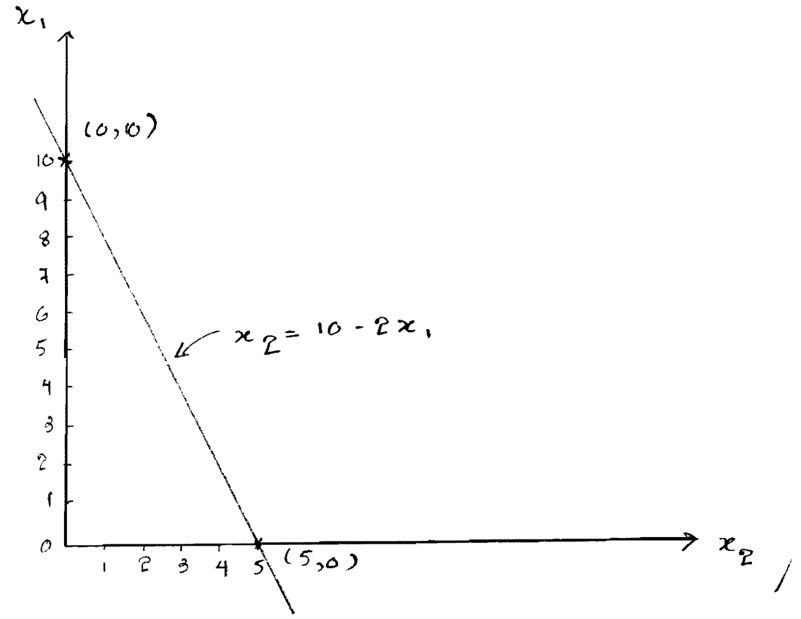
$$10 + (-10) - 2x_1 = 0 + (-10)$$

$$-2x_1 = -10$$

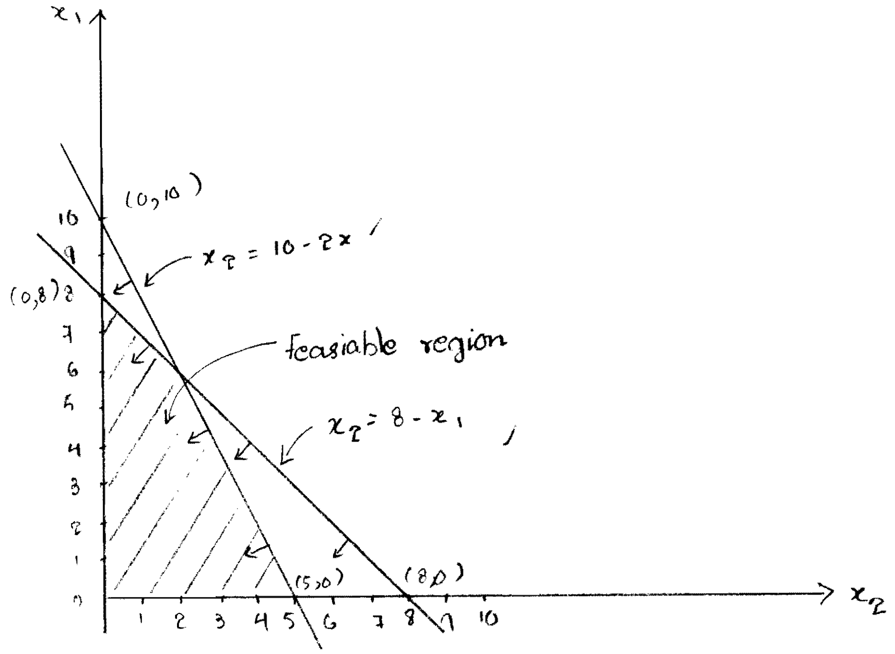
$$-2x_1 \times (-\tfrac{1}{2}) = -10 \times (-\tfrac{1}{2})$$

$$x_1 = 5$$

therefore , x intersection $= (5, 0)$



feasible region



150 − 9 = 141   + | [illegible]

Public Policy Modeling - Homework 3.

Question 1.

Maximize · $P = 3x_1 + 2x_2$

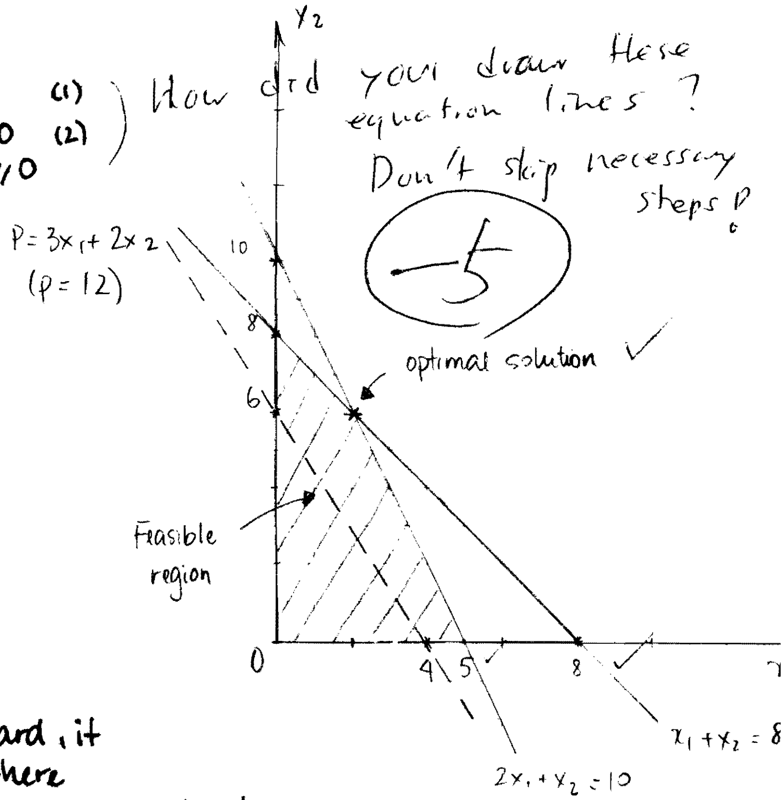
Subject to: $x_1 + x_2 \le 8$ (1)

$2x_1 + x_2 \le 10$ (2)

$x_1 \geq 0$, $x_2 \geq 0$

How did you draw these equation lines? Don't skip necessary steps!

−5



Objective function.

$P = 3x_1 + 2x_2$

$P - 3x_1 = 3x_1 - 3x_1 + 2x_2$

$2x_2 = P - 3x_1$

$\frac{1}{2} \cdot 2x_2 = \frac{1}{2}(P - 3x_1)$

$x_2 = -\frac{3}{2}x_1 + \frac{P}{2}$.

If $P = 12 \rightarrow x_2 = -\frac{3}{2}x_1 + \frac{12}{2}$

$x_2 = -\frac{3}{2}x_1 + 6$.

when we shift the line $P = 3x_1 + 2x_2$ upward, it will go through the optimal solution point where P is maximized, which is the intersection between two lines (1), (2)

→ solve for optimal solution.

$x_1 + x_2 = 8$ (1)

$2x_1 + x_2 = 10$ (2)

→ (2) − (1): $2x_1 + x_2 - x_1 - x_2 = 10 - 8$

$x_1 = 2$

Plug $x_1 = 2$ into (1): $2 + x_2 = 8$

$2 - 2 + x_2 = 8 - 2$

$x_2 = 6$.

→ optimal solution: production rate for product 1: 2 produc

" " " " 2: 6 "

Question 2:

(2.1). LP problem formulation.

→ Decision variables:

- x_1: number of full time consultants working from 8 am - 4pm (shift 1).
- x_2: " " " " " " " 12pm - 8pm (shift 2)
- x_3: " " " " " " " 4pm - midnight (shift 3). ✓
- x_4: number of part-time consultant working from 8am - 12pm ✓
- x_5: " " " " " " 12 pm - 4 pm ✓
- x_6: " " " " " " 4 pm - 8 pm ✓
- x_7: " " " " " " 8 pm - midnight. ✓

→ Objective function:

This is a minimization problem:

Minimize Cost $= (14 \times 8)x_1 + (14 \times 8)x_2 + (14 \times 8)x_3 + (12 \times 4)x_4 + (12 \times 4)x_5 + (12 \times 4)x_6 + (12 \times 4)x_7$

$= 112x_1 + 112x_2 + 112x_3 + 48x_4 + 48x_5 + 48x_6 + 48x_7$. ✓

→ Subject to:

- $x_1 + x_4 \geq 6$ ✓
  $x_1 + x_2 + x_5 \geq 8$ ✓
  $x_2 + x_3 + x_6 \geq 12$ ✓
  $x_3 + x_7 \geq 6$ ✓

- $x_1 \geq 2$
  $x_1 + x_2 \geq 2$
  $x_2 + x_3 \geq 2$
  $x_3 \geq 2$

- $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_5 \geq 0$, $x_6 \geq 0$, $x_7 \geq 0$.

(2.2) Solve the problem using Solver → see attachments.

(2.3) Divisibility is the most problematic assumption in this LP problem. Divisibility says that the legitimate value of decision variable are non-negative real numbers, which include fraction. However, in this case, the number of consultants working cannot take on fraction value. You cannot have 7/5 consultant working.

③

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Public Policy Modelling 2017**												
2	**Question 2 - Homework Assignment 3 (LP)**												
3	**ID: 1B6051**												
4	**Name: Nguyen Cam Linh**												
5													
6													
7				8AM-4PM	12PM-8PM	4PM-12AM	8AM-12PM	12PM-4PM	4PM-8PM	8PM-12AM			
8				Shift	Shift	Shift	Shift	Shift	Shift	Shift			
9			Cost per shift	$112	$112	$112	$48	$48	$48	$48			
10											LHS		RHS
11											Total		Minimum
12	Constraints		Time period				Shift work time period				Working		Needed
13			8AM-12PM	1	0	0	1	0	0	0	6	>=	6
14			12PM-4PM	1	1	0	0	1	0	0	8	>=	8
15			4PM-8PM	0	1	1	0	0	1	0	12	>=	12
16			8PM-12AM	0	0	1	0	0	0	1	6	>=	6
17			8AM-4PM	1	0	0	0	0	0	0	2	>=	2
18			12PM-8PM	1	1	0	0	0	0	0	2	>=	2
19			4PM-12AM	0	1	1	0	0	0	0	2	>=	2
20			8AM-12AM	0	0	1	0	0	0	0	2	>=	2
21													
22	Decision variables			x1	x2	x3	x4	x5	x6	x7			
23			No. consultant	2	0	2	4	6	10	4			
24													
25	Objective function		Total cost	[redacted]	MIN								
26													
27	Non-negativity												
28													
29													
30													

④

	A	B	C	D	E	F	G	H	I
1	**Public Policy Modelling 2017**								
2	**Question 2 - Homework Assignment 3 (LP)**								
3	**ID: 1B6051**								
4	**Name: Nguyen Cam Linh**								
5									
6									
7	**Microsoft Excel 14.0 Answer Report**								
8	**Worksheet: [LP Homework 3.xlsx]Question 3.16**								
9	**Report Created: 5/2/2017 10:18:50 AM**								
10	**Result: Solver found a solution. All Constraints and optimality conditions are satisfied.**								
11	**Solver Engine**								
12		Engine: Simplex LP							
13		Solution Time: 0.015 Seconds.							
14		Iterations: 13 Subproblems: 0							
15	**Solver Options**								
16		Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling							
17		Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegativ							
18									
19									
20	Objective Cell (Min)								
21		**Cell**	**Name**	**Original Value**	**Final Value**				
22		$D$25	Total cost x1	1600	1600				
23									
24									
25	Variable Cells								
26		**Cell**	**Name**	**Original Value**	**Final Value**	**Integer**			
27		$D$23	No. consultant x1	2	2	Contin			
28		$E$23	No. consultant x2	0	0	Contin			
29		$F$23	No. consultant x3	2	2	Contin			
30		$G$23	No. consultant x4	4	4	Contin			
31		$H$23	No. consultant x5	6	6	Contin			
32		$I$23	No. consultant x6	10	10	Contin			
33		$J$23	No. consultant x7	4	4	Contin			
34									
35									
36	Constraints								
37		**Cell**	**Name**	**Cell Value**	**Formula**	**Status**	**Slack**		
38		$K$13	8AM-12PM Working	6	$K$13>=$M$13	Binding	0		
39		$K$14	12PM-4PM Working	8	$K$14>=$M$14	Binding	0		
40		$K$15	4PM-8PM Working	12	$K$15>=$M$15	Binding	0		
41		$K$16	8PM-12AM Working	6	$K$16>=$M$16	Binding	0		
42		$K$17	8AM-4PM Working	2	$K$17>=$M$17	Binding	0		
43		$K$18	12PM-8PM Working	2	$K$18>=$M$18	Binding	0		
44		$K$19	4PM-12AM Working	2	$K$19>=$M$19	Binding	0		
45		$K$20	8AM-12AM Working	2	$K$20>=$M$20	Binding	0		

Question 3:

(3.1) Check unit of measurement.

For the promotional campaign, TV has its unit of # spot ; M and SS have their units of # of ads ; their coefficients are measured in thousand viewing / spot and thousand viewing / ad respectively

→ objective function:

Max exposure (thousand viewing) = 1,300 TV (thousand viewing / spot × spot) + 600 M (thousand viewing / ad × ad) + 500 SS (thousand viewing / ad × ad).

→ Resources constraints: coefficients are measured in thousand $ / spot or ad.

300 TV (thousand $ / spot × spot) + 150 M (thousand $ / ad × ad) + 100 SS (thousand $ / ad × ad) ≤ 4,000 (thousand $)

90 TV (thousand $ / spot × spot) + 30 M (thousand $ / ad × ad) + 40 SS (thousand $ / ad × ad) ≤ 1,000 (thousand $)

TV (spot) ≤ 5 (spot)

→ Benefit constraints:

For these constraints, the coefficients are measured in million children and million parents / spot or ad.

1.2 TV (million children / spot × spot) + 0.1 M (million children / ad × ad) ≥ 5 (million children).

0.5 TV (million parents / spot × spot) + 0.2 M (million parents / ad × ad) + 0.2 SS (million parents / ad × ad) ≥ 5 (million parents).

→ Fixed requirement constraints.

40 M (thousand $ / ad × ad) + 120 SS (thousand $ / ad × ad) = 1,490 (thousand $)

(3.2) Solve the problem using Solver → See attachments.

(3.3) Calculated optimal value.

→ Optimal solutions:
- TV : 3 spots.
- M : 14 ads
- SS : 7.75 ads.

→ optimal value: C = 1,300 TV + 600 M + 500 SS
= 1,300 × 3 + 600 × 14 + 500 × 7.75
= 16,175. (thousand $).

(−2)

(3.4) Surplus is for a "≥ type" constraint → in this case, surplus will happen to the benefit constraints.

We can see that constraint for the number of parents of young children reached has a surplus of 0.85 million parents. ✓

Surplus = LHS − RHS = (0.5 × 3 + 0.2 × 14 + 0.2 × 7.75) − 5 = 0.85. ✓

(3.5) Objective coefficient : 500 thousand viewings.
Allowable increase : 577.7778 thousand viewings.
Allowable decrease : 1E+30 " "

→ the coefficient for Sunday supplement ad (the number of viewing) can change from negative infinity "−∞" (500 − 1E+30) to 1,077.7778 viewing without changing the optimal solution.

(3.6) Complementary slackness says that if a constraint is binding and has 0 slack or surplus then its shadow price is not 0 and vice versa.

For example:
→ From Answer Report, the planning budget constraint is binding and has 0 slack; on Sensitivity Report, this constraint has a shadow price of 35.4545 (not = 0). ✓

→ On sensitivity report, young children constraint has a shadow price of −1575.75 (not = 0); then on the Answer report, this constraint is binding and has 0 slack. ✓

(3.7) The largest shadow price is for constraint on planning budget = 35.4545.

→ When the RHS of this constraint increases by one unit in the range of (1,000 − 85 ; 1000 + 22.5) = [915, 1,022.5], then the number of exposures will ~~be~~ increased by 35.4545 unit. — ambiguous word here
thousand exposures.

(−2)

(3.8) The allowable range for the RHS of the coupon budget :

(1,490 − 90 ; 1,490 + 385) = [1400, 1875]. (unit in thousands $)

The increase of 300,000 $ is still within the allowable range → the shadow price for the constraint remains valid (= −7.6515) → the optimal value will decrease and the optimal solution will change. ✓

(−2)

how much?

① (−7.6515 * 300) = −2295.45 thousand exposures

② optimal solution may change

③ shadow price remains valid.

→ not a polite answer!

	A	B	C	D	E	F	G	H	I
1	**Public Policy Modelling 2017**								
2	**Question 3 - Homework Assignment 3 (LP)**								
3	**ID: 1B6051**								
4	**Name: Nguyen Cam Linh**								
5									
6									
7	**Super Grain Corp. Advertising Mix Problem**								
8									
9									
10									
11				TV spots	Magazine Ads	SS Ads			
12		Exposure per Ad		1300	600	500			
13		(thousand $)							
14									
15					Cost per Ad (thousands $)		Budget spent		Budget available
16		Ad Budget		300	150	100	3775	<=	4000
17		Planning Budget		90	30	40	1000	<=	1000
18									
19					Number reached per Ad (million people)		Total reached		Minimum acceptable
20		Young children		1.2	0.1	0	5	>=	5
21		Parents of young children		0.5	0.2	0.2	5.85	>=	5
22									
23				TV spots	Magazine Ads	SS Ads	Total redeemed		
24		Coupon Redemption		0	40	120	1490	=	1490
25		per Ad (thousands$)							
26									Total exposure
27				TV spots	Magazine Ads	SS Ads			(thousands $)
28		Number of Ads		3	14	7.75			[redacted]
29				<=					
30		Maximum TV Spots		5					

Public Policy Modelling 2017
Question 3 - Homework Assignment 3 (LP)
ID: 1B6051
Name: Nguyen Cam Linh

Microsoft Excel 14.0 Answer Report
Worksheet: [LP Homework 3.xlsx]Super Grain
Report Created: 5/2/2017 10:08:47 AM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 5 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$I$28	Number of Ads (thousands $)	16175	16175

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$28	Number of Ads TV spots	3	3	Contin
$E$28	Number of Ads Magazine Ads	14	14	Contin
$F$28	Number of Ads SS Ads	7.75	7.75	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$G$16	Ad Budget Budget spent	3775	$G$16<=$I$16	Not Binding	225
$G$17	Planning Budget Budget spent	1000	$G$17<=$I$17	Binding	0
$G$20	Young children Total reached	5	$G$20>=$I$20	Binding	0
$G$21	Parents of young children Total reached	5.85	$G$21>=$I$21	Not Binding	0.85
$G$24	Coupon Redemption Total redeemed	1490	$G$24=$I$24	Binding	0
$D$28	Number of Ads TV spots	3	$D$28<=$D$30	Not Binding	2

	A	B	C	D	E	F	G	H
1	**Public Policy Modelling 2017**							
2	**Question 3 - Homework Assignment 3 (LP)**							
3	**ID: 1B6051**							
4	**Name: Nguyen Cam Linh**							
5								
6								
7	**Microsoft Excel 14.0 Sensitivity Report**							
8	**Worksheet: [LP Homework 3.xlsx]Super Grain**							
9	**Report Created: 5/2/2017 10:08:47 AM**							
10								
11								
12	Variable Cells							
13				**Final**	**Reduced**	**Objective**	**Allowable**	**Allowable**
14		**Cell**	**Name**	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
15		$D$28	Number of Ads TV spots	3	0	1300	1040	1E+30
16		$E$28	Number of Ads Magazine Ads	14	0	600	1E+30	192.5925926
17		$F$28	Number of Ads SS Ads	7.75	0	500	577.7777778	1E+30
18								
19	Constraints							
20				**Final**	**Shadow**	**Constraint**	**Allowable**	**Allowable**
21		**Cell**	**Name**	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
22		$G$16	Ad Budget Budget spent	3775	0	4000	1E+30	225
23		$G$17	Planning Budget Budget spent	1000	35.45454545	1000	22.5	85
24		$G$20	Young children Total reached	5	-1575.757576	5	1.32	0.45
25		$G$21	Parents of young children Total reached	5.85	0	5	0.85	1E+30
26		$G$24	Coupon Redemption Total redeemed	1490	-7.651515152	1490	385	90