1

Homework 3

152 − 14 = 136

Q1.

	Table	chair	amount of the avalable resources
material	50	25	2500
labor	6	6	480
net profit	400	100	
(# of Sales) = requirements of producing	α	2α	

+1 ex (what-pound in each month)

1.1) Decision variables are defined as
T (the number of tables to produce per month) and /
C (the number of chairs to produce per month). /

· Objective function

Maximize Profit = 400 T + 100 C /

· Constraints.

material	:	50 T + 25 C ≦ 2500 /
labor	:	6 T + 6 C ≦ 480 /
required # of producing	:	2 T − C ≦ 0 / (2T ≦ C was reorganized)

· Nonnegativity /

T ≧ 0
C ≧ 0

1.2) Check the unit of measurement

· Objective function

Max Profit = 400 T + 100 C (Math expression) /
$/ Month = ($/EA) × (EA/Month) + ($/EA) × (EA/Month)

T and C respectively have their units of EA/Month (the number of tables and chairs to produce per month). Their coefficients are measured in the unit price ($/EA). The result of objective function is $ (amount of earnings per month in USD).
In this objective function, has reasonable units.

Total profitだけ($/100)単位が可能??

- Constraints

material : $50T + 25C \leqq 2500$

(pounds / EA) × (EA / Month) + (pounds / EA) × (EA / Month) ≦ (pounds / Month)

labor : $6T + 6C \leqq 480$

(hours / EA) × (EA / Month) + (hours / EA) × (EA / Month) ≦ (hours / Month)

required # of producing : $2T - C \leqq 0$

(the number of tables / EA) × (EA / Month) − (the number of chairs / EA) × (EA / Month) ≦ (the number of tables / chairs)

1.3)

Objective function : $P = 400T + 100C$

Constraints :

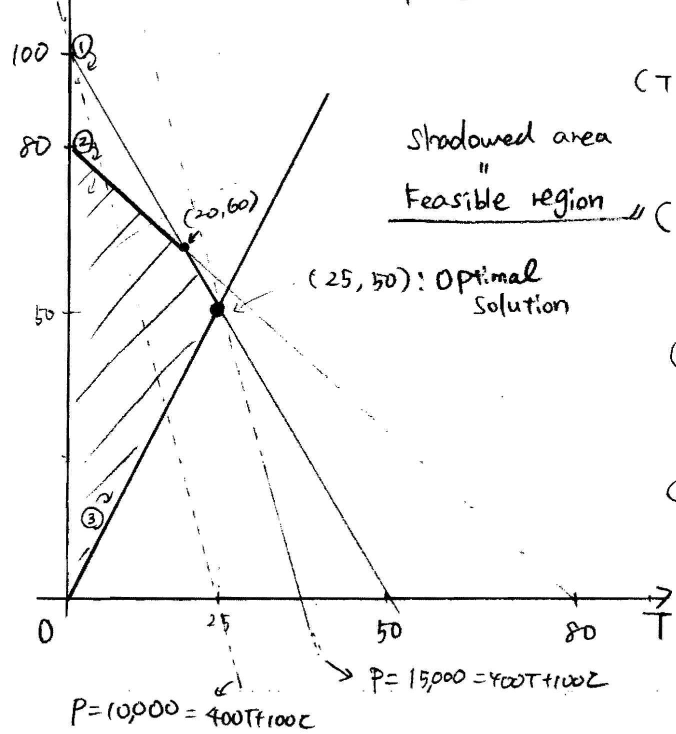
$50T + 25C \leqq 2500$ ···①

$6T + 6C \leqq 480$ ···②

$2T - C \leqq 0$ ···③

$T \geqq 0$

$C \geqq 0$



① ⇔ $25C \leqq -50T + 2500$

(Transfer 50T to RHS)

$\frac{25}{25}C \leqq -\frac{50}{25}T + \frac{2500}{25}$

$C \leqq -2T + 100$

(cancellation rule by multiplication (25))

② ⇔ $6C \leqq -6T + 480$

(Transfer 6T to RHS)

$\frac{6}{6}C \leqq -\frac{6}{6}T + \frac{480}{6}$

$C \leqq -T + 80$

(cancellation rule by multiplication (6))

③ ⇔ $-C \leqq -2T$

(Transfer 2T to RHS)

$-1 \times (-C) \geqq -1 \times (-2)T$

$C \geqq 2T$

(cancellation rule by multiplication (−1))

	A	B	C	D	E	F	G	H
1	**Oak Works**							
2	Hillier and Hillier (2014) Q2.20 p.57-58							
3								
4	**Raw data**		x1	x2				
5		Profit	400	100				
6								
7								
8	**Constraints**					RHS		
9		Constraint 1	50	25	<=	2500		
10		Constraint 2	6	6	<=	480		
11		Constraint 3	2	-1	<=	0		
12								
13	**Decision**		x1	x2		Total Cost		
14			25	50		15000		
15								
16		Constraint 1	2500		<=	2500		
17		Constraint 2	450		<=	480		
18		Constraint 3	0		<=	0		
19								
20	**Nonnegativity**							
21								
22								
23	Objective Cell (Max)							
24		Cell	Name	Original Valu	Final Value			
25		$F$14	Total Cost	0	15000			
26								
27								
28	Variable Cells							
29		Cell	Name	Original Valu	Final Value	Integer		
30		$C$14	x1	0	25	Contin		
31		$D$14	x2	0	50	Contin		
32								

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$C$16	Constraint 1 x1	2500	$C$16<=$F$16	Binding	0
$C$17	Constraint 2 x1	450	$C$17<=$F$17	Not Binding	30
$C$18	Constraint 3 x1	0	$C$18<=$F$18	Binding	0

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$14	x1	25	0	400	1E+30	200
$D$14	x2	50	0	100	100	300

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$16	Constraint 1 x1	2500	6	2500	166.6667	2500
$C$17	Constraint 2 x1	450	0	480	1E+30	30
$C$18	Constraint 3 x1	0	50	0	100	20

Question 2:

- Decision Variables
  - X_{1T} : number of shipments per month produced by plant 1 and received by Tokyo ✓
  - X_{1NA} : " " " " " by plant 1 " " by Nagano
  - X_{1NI} : " " " " " by plant 1 " " by Niigata
  - X_{1k} : " " " " " by plant 1 " " by Kumamoto
  - X_{2T} : " " " " " by plant 2 " " by Tokyo
  - X_{2NA} : " " " " " by plant 2 " " by Nagano
  - X_{2NI} : " " " " " by plant 2 " " by Niigata
  - X_{2k} : " " " " " by plant 2 " " by Kumamoto
  - X_{3T} : " " " " " by plant 3 " " by Tokyo
  - X_{3NA} : " " " " " by plant 3 " " by Nagano
  - X_{3NI} : " " " " " by plant 3 " " by Niigata
  - X_{3k} : " " " " " by plant 3 " " by Kumamoto
  - X_{4T} : " " " " " by plant 4 " " by Tokyo
  - X_{4NA} : " " " " " by plant 4 " " by Nagano
  - X_{4NI} : " " " " " by plant 4 " " by Niigata
  - X_{4k} : number of shipments per month produced by plant 4 " " by Kumamoto

- LP formulation:

We are facing a Minimization Problem

Minimize $Cost = 500X_{1T} + 600X_{1NA} + 400X_{1NI} + 200X_{1k} + 200X_{2T} + 900X_{2NA} + 100X_{2NI} + 300X_{2k} + 300X_{3T} + 400X_{3NA} + 200X_{3NI} + 100X_{3k} + 200X_{4T} + 100X_{4NA} + 300X_{4NI} + 200X_{4k}$ ✓

Subject to :

1. $X_{1T} + X_{2T} + X_{3T} + X_{4T} = 20$
2. $X_{1NA} + X_{2NA} + X_{3NA} + X_{4NA} = 10$

$X_{1NI} + X_{2NI} + X_{3NI} + X_{4NI} = 10$

$X_{1k} + X_{2k} + X_{3k} + X_{4k} = 20$

} Shipment Reception Constraint ✓

and

$X_{1T} + X_{1NA} + X_{1NI} + X_{1k} = 10$

$X_{2T} + X_{2NA} + X_{2NI} + X_{2k} = 20$

$X_{3T} + X_{3NA} + X_{3NI} + X_{3k} = 20$

$X_{4T} + X_{4NA} + X_{4NI} + X_{4k} = 10$

} Shipment Production Constraint

)

8

and

$X_{1T} \geq 0$, $X_{1NA} \geq 0$, $X_{1NI} \geq 0$, $X_{1K} \geq 0$, $X_{2T} \geq 0$, $X_{2NA} \geq 0$, $X_{2NI} \geq 0$

$X_{2K} \geq 0$, $X_{3T} \geq 0$, $X_{3NA} \geq 0$, $X_{3NI} \geq 0$, $X_{3K} \geq 0$, $X_{4T} \geq 0$, $X_{4NA} \geq 0$, $X_{4NI} \geq 0$

$X_{4K} \geq 0$ ✓

Q 2.2 : Optimal Solution

After Solving this LP problem using Excel Solver we found that the optimal Solution is : -

- To make 10 shipment using the plant 1 and send it to Kumamoto (per month)
- To make 20 shipment using plant 2 and send 10 to Tokyo and Send 10 to Niigata (per month)
- To make 20 shipment using plant 3 and send 10 to Tokyo and Send 10 to Kumamoto (per month)
- To make 10 shipment using plant 4 and send it to Nagano (per month)

And the optimal Value (Minimum shipping cost) is $10 000 per month.

	A	B	C	D	E	F	G	H	I
1	**Childfair Company**								
2	Hillier and Hillier (2014) Q3.28 p.112								
3									
4	**Raw Data**	**Distance**							
5									
6		**Shipping Cost**							
7		**(USD)**	Tokyo	Nagano	Niigata	Kumamoto	Shipped		
8		Plant 1	$500	$600	$400	$200	10		
9		Plant 2	$200	$900	$100	$300	20		
10		Plant 3	$300	$400	$200	$100	20		
11		Plant 4	$200	$100	$300	$200	10		
12		Delivered	20	10	10	20			
13									
14	**ecision variables**		x11	x12	x13	x14			
15			x21	x22	x23	x24			
16			x31	x32	x33	x34			
17									
18	**bjective functio**	Total Cost	$10,000						
19									
20									
21	**Constraints**	**Units Shipped**	Tokyo	Nagano	Niigata	Kumamoto	Shipped		Limit
22		Plant 1	0	0	0	10	10	=	10
23		Plant 2	10	0	10	0	20	=	20
24		Plant 3	10	0	0	10	20		20
25		Plant 4	0	10	0	0	10		10
26		Delivered	20	10	10	20			
27			=	=	=				
28		Limit	20	10	10	20			
29									
30	**Nonnegativity**								
31									
32									
33	Objective Cell (Min)								
34		Cell	Name	)riginal Valu	Final Value				
35		$C$18	Total Cost x:	$10,000	$10,000				
36									
37									
38	Variable Cells								
39		Cell	Name	)riginal Valu	Final Value	Integer			
40		$C$22	Plant 1 Toky	0	0	Contin			
41		$D$22	Plant 1 Naga	0	0	Contin			
42		$E$22	Plant 1 Niiga	0	0	Contin			
43		$F$22	Plant 1 Kuma	10	10	Contin			
44		$C$23	Plant 2 Toky	10	10	Contin			
45		$D$23	Plant 2 Naga	0	0	Contin			
46		$E$23	Plant 2 Niiga	10	10	Contin			
47		$F$23	Plant 2 Kuma	0	0	Contin			
48		$C$24	Plant 3 Toky	10	10	Contin			
49		$D$24	Plant 3 Naga	0	0	Contin			
50		$E$24	Plant 3 Niiga	0	0	Contin			
51		$F$24	Plant 3 Kuma	10	10	Contin			

	A	B	C	D	E	F	G	H	I
52		$C$25	Plant 4 Toky	0	0	Contin			
53		$D$25	Plant 4 Naga	10	10	Contin			
54		$E$25	Plant 4 Niiga	0	0	Contin			
55		$F$25	Plant 4 Kuma	0	0	Contin			
56									
57									
58	Constraints								
59		Cell	Name	Cell Value	Formula	Status	Slack		
60		$C$26	Delivered Tol	20	$C$26=$(	Binding	0		
61		$D$26	Delivered Na	10	$D$26=$I	Binding	0		
62		$E$26	Delivered Nii	10	$E$26=$E	Binding	0		
63		$F$26	Delivered Ku	20	$F$26=$F	Binding	0		
64		$G$22	Plant 1 Shipp	10	$G$22=$I	Binding	0		
65		$G$23	Plant 2 Shipp	20	$G$23=$I	Binding	0		
66		$G$24	Plant 3 Shipp	20	$G$24=$I	Binding	0		
67		$G$25	Plant 4 Shipp	10	$G$25=$I	Binding	0		

Question 3 :

Q3.1. LP Problem formulation :

▸ Decision variables

- X_1 : Unit of Product 1 to produce per week.
- X_2 : Unit of Product 2 to produce per week.
- X_3 : Unit of Product 3 to produce per week.

▸ Objective function

We are facing a Maximization problem

$$\text{Max Profit} = 50X_1 + 20X_2 + 10X_3$$ ✓

▸ Constraints

1. $9X_1 + 3X_2 + 5X_3 \leq 550$ ✓
2. $5X_1 + 4X_2 \leq 350$ ✓
3. $3X_1 + 2X_3 \leq 150$ ✓

(Production constraints)

4. $X_3 \leq 20$ (Market constraint) ✓

· Non-negativity : $X_1 \geq 0$, $X_2 \geq 0$ and $X_3 \geq 0$ ✓

Q3.2 Checking Unit of measurement

▸ Objective function Profit $= 50X_1 + 20X_2 + 10X_3$ (math expression)

$\$/\text{week} = \$/EA \times EA/\text{week} + \$/EA \times EA/\text{week}$

$\$/\text{week} = \$/\text{week} + \$/\text{week}$

$\$/\text{week} = \$/\text{week}$

▸ Production Constraints $9X_1 + 3X_2 + 5X_2 \leq 550$

$5X_1 + 4X_2 \leq 350$ (math expressions)

$3X_1 + 2X_3 \leq 150$

Optimal solutions :

Producing the ff will give the maximized profit of $3,000:

3.4) $X_1 = 50$ units Product 1

$X_2 = 25$ units Product 2

$X_3 = 0$ units Product 3

3.5) Product 2 Objective coefficient.

From the sensitivity report in the results of Q3.3, the objective coefficient of 20 for product 2 can range from [0,28]. This means that the unit profit for Product 2 can range from [0,28] without affecting the optimal solutions / the optimal amounts of products 1, 2 and 3 to be produced. But, since the unit profit will change, obviously, the maximized profit will also be affected.

3.6) Non-zero reduced cost

X_3 = Product 3 has non-zero reduced cost of $-6.67. It is non-zero bec. we are not producing any X_3 ($X_3 = 0$). This value of $6.67 is the amount that will be deducted from the maximized profit if we force to produce 1 unit of Product 3.

3.7) Non-binding Constraints

* Milling Machine Used Hrs : Slack = |RHS - LHS| = |550 - 525| = 25

  from Constraint 1 : $9x_1 + 3x_2 + 5x_3 \leq 550$

  : $9(50) + 3(25) + 5(0) \leq 550$

  : $525 \leq 550$.

  → This implies that milling machine is actually underutilized as a resource.

* Unit X_3 Produced

  from constraint 4 : $X_3 \leq 20 \Rightarrow 0 \leq 20$

  Slack = |RHS - LHS|

  = |20 - 0| = 20

3.8) Largest Shadow price.

The constraint for Grinder has the largest Shadow price = 8.33. It means that, for every additional 1 hr available in grinder 3, profit will increase by $8.33. And for every 1 hr that is deducted from the 150 hrs available in grinder 3 (eg. problem in operation, machine shutdown, etc.), the profit will decrease by $8.33. This value of $8.33 is valid from the range of [0, 164].

3.9) Given:
- cost of increasing 1 milling machine hr/week = $2
- " " " 1 lathe hr/week = $3
- " " " 1 grinder hr/week = $7
- $15 to invest → cost ≤ 15

From sensitivity Report on Q 3.3

Constraint	Shadow price ($/hr)	Constraint RH Side (hrs)	Allowable Range	Status
milling machine	0	550	[525, ∞]	Not binding
Lathe	5	350	[250, 383]	binding
Grinder	8.3	150	[0, 164]	binding.

* Don't invest in milling machine. It is useless (not-binding) and an underutilized resource.

* If we increase 2hrs in grinder, Marginal Profit = (Shadow Price * hrs) - cost of Increase.

marginal Profit = Marginal Increase - cost of Increase

$$MP = \left(8.3\ \frac{\$}{hr}\right)(2\ hrs) - \left(7\ \frac{\$}{hr}\right)(2\ hrs)$$

$$MP = \$2.6$$

* If we increase 5hrs in lathe, $MP = \left(5\ \frac{\$}{hr}\right)(5\ hrs) - \left(3\ \frac{\$}{hr}\right)(5\ hrs)$

$MP = \$10$

good

* If we increase 2hrs in lathe, and 1 hr in grinder:

$$MP = \underbrace{\left(5\ \frac{\$}{hr}\right)(2\ hrs) - \left(3\ \frac{\$}{hr}\right)(2\ hrs)}_{\text{lathe}} + \underbrace{\left(8.3\ \frac{\$}{hr}\right)(1\ hr) - \left(7\ \frac{\$}{hr}\right)(1\ hr)}_{\text{grinder}}$$

$= \$4 + \$1.3$

$MP = \$5.3$.

Recommendation: Use $15 to invest in Lathe and increase its available hrs by 5 hrs. It will give the highest marginal profit of $10.

Correct answer!

3.10) LP formulation Limitation

- Divisibility assumption. Adding 5 more hrs to lathe may lead to an "unideal" optimal solution (non-integer) that would not make sense in reality. In fact if we rerun the LP with the new constraint of 355 hrs (vs 350 hrs), we will get $x_2 = 26.25$. 26.25 units of Product 2 is absurd.

Omega Manufacturing Company

Hillier and Hillier (2014) Q 3.6 p.106-107

	Product 1	Product 2	Product 3			
Profit	50	20	10			
	Cumulative Capital Required ($millions)			Consumption		Limit
Miling machine	9	3	5	525	<=	550
Lathe	5	4	0	350	<=	350
Grinder	3	0	2	150	<=	150
	0	0	1	0	<=	20

	Product 1	Product 2	Product 3	Profit
Decision variables	50.0	25.0	0.0	3000.00

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$H$14	Decision variables F	3000.00	3000.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$14	Decision variables F	50.0	50.0	Contin
$D$14	Decision variables F	25.0	25.0	Contin
$E$14	Decision variables F	0.0	0.0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$8	Miling machine Cons	525	$F$8<=$H$8	Not Binding	25
$F$9	Lathe Consumption	350	$F$9<=$H$9	Binding	0

	A	B	C	D	E	F	G	H
33		$F$10	Grinder Consumptio	150	$F$10<=H1C	Binding	0	
34		$F$11	Consumption	0	$F$11<=$H$11	Not Binding	20	
35								
36	Variable Cells							

		Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
39		$C$14	Decision variables F	50	0	50	1E+30	10
40		$D$14	Decision variables F	25	0	20	8	20
41		$E$14	Decision variables F	0	-6.66666667	10	6.666666667	1E+30
42								
43	Constraints							

		Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
46		$F$8	Miling machine Cons	525	0	550	1E+30	25
47		$F$9	Lathe Consumption	350	5	350	33.33333333	100
48		$F$10	Grinder Consumptio	150	8.333333333	150	14.28571429	150
49		$F$11	Consumption	0	0	20	1E+30	20