INTERNATIONAL UNIVERSITY OF JAPAN

Public Management and Policy Analysis Program
Graduate School of International Relations

DCC5350 (2 Credits)
Public Policy Modeling
Spring 2017

Midterm Exam (100 points)

Instruction: Please write down your student ID and name at the top of your answer. You MUST always show necessary computation and your reasoning as clearly as possible. When handling rational numbers (e.g., 3.1415), use at least three digits below the decimal point.

Question 1 (40 points) Read Question 9.17 on page 373. Skip questions *a-k*.

1.1 (5 points) Make a decision using EMV.
1.2 (5 points) Construct the regret table.
1.3 (5 points) Make a decision using EOL.
1.4 (5 points) Calculate the expected value under perfect information. Draw a decision tree under perfect information.
1.5 (5 points) Calculate posterior probabilities. Use conditional probabilities of P(Predicted S1|S1)=P(PS1|S1)=.6 and P(PS2|S2)=.8.
1.6 (5 points) Calculate the expected value under imperfect information.
1.7 (5 points) Draw a decision tree under this imperfect information.
1.8 (5 points) Would you like to pay for the research? Why and why not?

Question 2 (10 points) Look at Q3.17* (Medequip Company) on page 109 and formulate this transportation LP problem. Skip questions *a-c*. Use x_{ij} for decision variables and "=" for inequality of constraints. First, define the decision variables clearly.

Question 3 (50 points) Look at Question 5.7 (University Ceramics) on page 183. You may skip the description below the raw data table (not Excel worksheet and sensitivity report).

3.1 (5 points) Formulate this LP problem completely. Use x_i for decision variables.
3.2 (5 points) Check the unit of measurement of the objective function and constraints. Ignore the first constraint.
3.3 (5 points) Report binding constraints and non-zero slack, if any. Show me your reasoning.
3.4 (5 points) Report a non-zero reduced cost and then interpret it in two ways.
3.5 (10 points) If the coefficient of the plate decision variable increases by $2.00, what will be the optimal solution and optimal value (of the objective function)? Show me your reasoning.
3.6 (5 points) Explain the meaning of the largest shadow price substantively and completely.
3.7 (10 points) Given the information above, what would you suggest (advise) if your boss wants to increase RHS of constraint 1 to 3,000?
3.8 (5 points) Among certainty, divisibility, proportionality, additivity, and homogeneity, which assumption do you think is most problematic in this LP formulation? And why?

End of the midterm exam.

Question -1 1010 - 17 = A3

$P(S_1) = 0.4$, $P(S_2) = 0.6$

(1.1) Make decision using EMV + 3

For A_1

$$EMV = \sum (\text{prior probability})(\text{payoff})$$
$$= 0.4 \times 400 + 0.6(-100)$$
$$= 100$$

For A_2

$$EMV = 0.4 \times 0 + 0.6 \times 100$$
$$= 60$$

Therefore, I will choose A_1 because it has largest expected monetary value ($100 > 60$)

(1.2) Construct the regret table

Regret	state of nature	
	S_1	S_2
A_1	0	200
A_2	400	0

$$\text{Regret}(A_1/S_1) = \text{Max}(400, 0) - 400 = 400 - 400 = 0$$
$$\text{Regret}(A_2/S_1) = \text{Max}(400, 0) - 0 = 400 - 0 = 400$$
$$\text{Regret}(A_1/S_2) = \text{Max}(-100, 100) - (-100) = 100 + 100 = 200$$
$$\text{Regret}(A_2/S_2) = \text{Max}(-100, 100) - 100 = 100 - 100 = 0$$

(1.3) decision using EOL

For A_1 $EOL = 0 \times 0.4 + 200 \times 0.6 = 120$

For A_2 $EOL = 400 \times 0.4 + 0 \times 0.6 = 160$

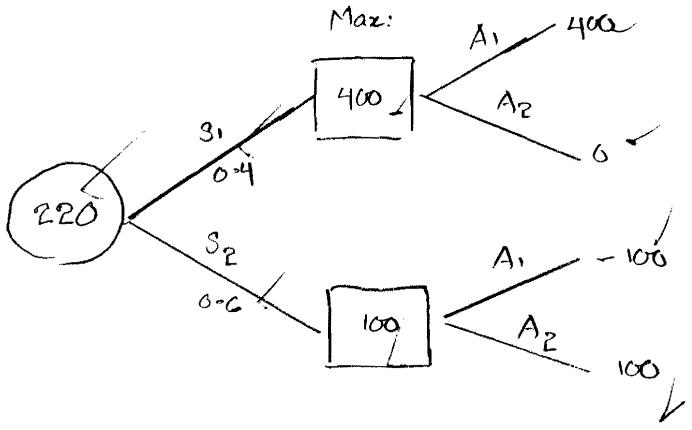
Therefore, I will choose A_1 because it has smallest expected opportunity loss ($120 < 160$)

(1.4) the expected value under perfect information

EV (under perfect info:) = Max (400 , 0) $\cancel{\times 0.4 + \text{Max}(-100, 100) \times 0.}$

= 400 × 0.4 + 100 × 0.6

= 220



(1.5) posterior probabilities

p (Predicted S_1 / S_1) = $P(PS_1/S_1) = 0.6$, $P(PS_2/S_2) = 0.8$

$P(PS_2/S_1) = 0.4$, $P(PS_1/S_2) = 0.2$

$P(S_1) = 0.4$, $P(S_2) = 0.6$

1st stage joint probabilities

$P(PS_1 \,\&\, S_1) = P(PS_1/S_1) \times P(S_1) = 0.6 \times 0.4 = 0.24$

$P(PS_2 \,\&\, S_1) = P(PS_2/S_1) \times P(S_1) = 0.4 \times 0.4 = 0.16$

$P(PS_1 \,\&\, S_2) = P(PS_1/S_2) \times P(S_2) = 0.2 \times 0.6 = 0.12$

$P(PS_2 \,\&\, S_2) = P(PS_2/S_2) \times P(S_2) = 0.8 \times 0.6 = 0.48$

2nd stage marginal probabilities

$P(PS_1) = P(PS_1 \,\&\, S_1) + P(PS_1 \,\&\, S_2)$

$= 0.24 + 0.12$

$= 0.36$

$P(PS_2) = P(PS_2 \,\&\, S_1) + P(PS_2 \,\&\, S_2)$

$= 0.16 + 0.48$

$= 0.64$

3rd stage posterior probabilities

$$P(S_1/PS_1) = \frac{P(S_1 \,\&\, PS_1)}{P(PS_1)} = \frac{0.24}{0.36} = 0.667$$

$$P(S_1/PS_2) = \frac{P(S_1 \,\&\, PS_2)}{P(PS_2)} = \frac{0.16}{0.64} = 0.25$$

+2

$$P(S_2/PS_1) = \frac{P(S_2 \,\&\, PS_1)}{P(PS_1)} = \frac{0.12}{0.36} = 0.333$$

$$P(S_2/PS_2) = \frac{P(S_2 \,\&\, PS_2)}{P(PS_2)} = \frac{0.48}{0.64} = 0.75$$

(1.6) EV (under Imperfect Information)

$$EV(A_1/PS_1) = P(S_1/PS_1) \times 400 + P(S_2/PS_1) \times (-100)$$
$$= 0.667 \times 400 + 0.333 \times (-100)$$
$$= 233.5$$

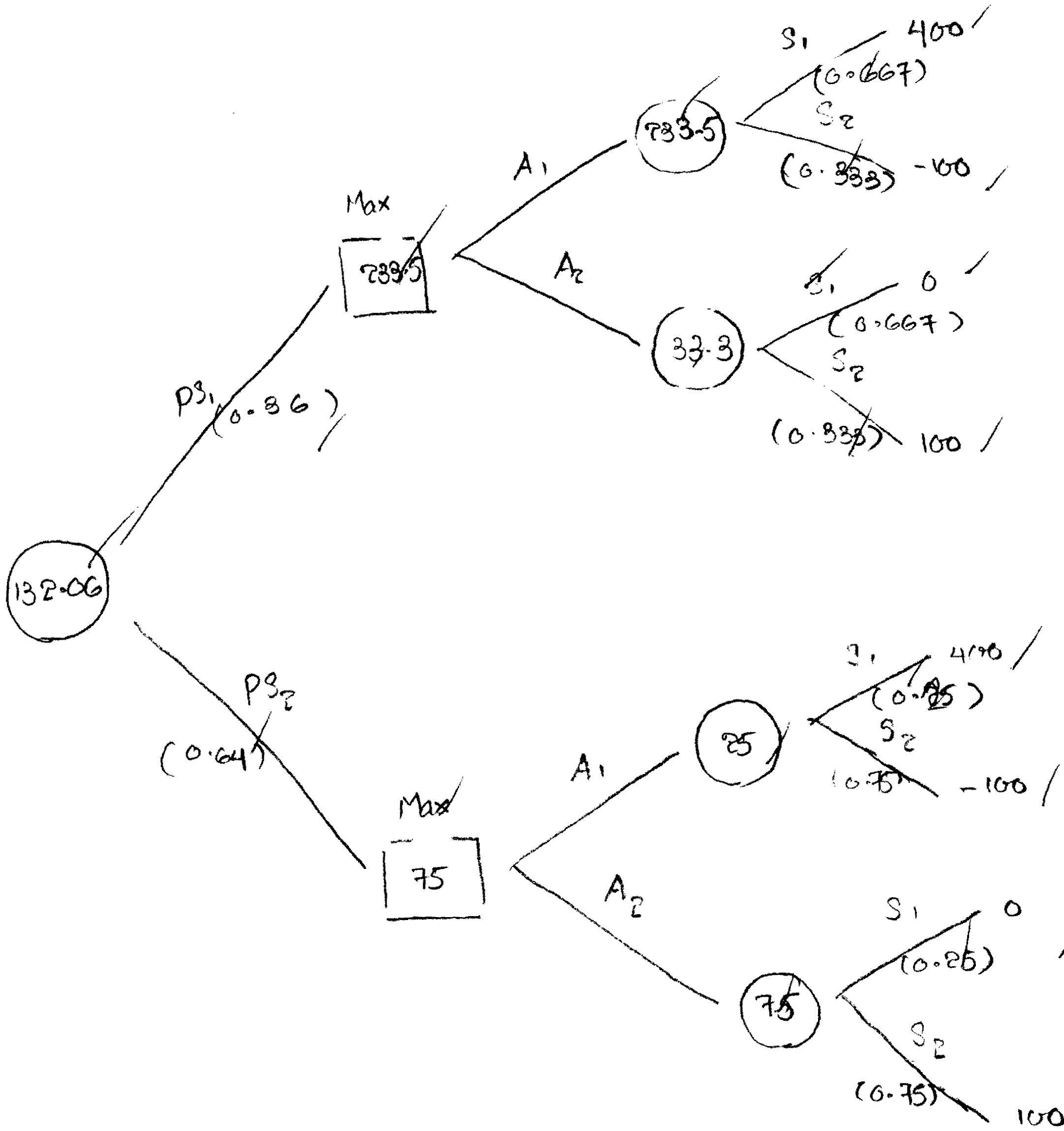
$$EV(A_2/PS_1) = P(S_1/PS_1) \times 0 + P(S_2/PS_1) \times 100$$
$$= 0.667 \times 0 + 0.333 \times 100$$
$$= 33.3$$

$$EV(A_1/PS_2) = P(S_1/PS_2) \times 400 + P(S_2/PS_2) \times (-100)$$
$$= 0.25 \times 400 + 0.75 \times (-100)$$
$$= 25$$

$$EV(A_2/PS_2) = P(S_1/PS_2) \times 0 + P(S_2/PS_2) \times 100$$
$$= 0.25 \times 0 + 0.75 \times 100$$
$$= 75$$

$$EV(\text{under imperfect info.}) = P(PS_1) \times Max(233.5, 33.3) + P(PS_2) \times Max(25, 75)$$
$$= 0.36 \times 233.5 + 0.64 \times 75$$
$$= 132.06$$

(1.7) decision tree under imperfect info:



1.8 No, I will not pay for the research because my EVII is only 32.06 ($). It means I want to pay under the price of 32.06 $. The calculation is as the following.

EVII = EV (under imperfect info) − EMV

= 132.06 − 100

= 32.06 ($)

Therefore, I will not pay for reasearch because

($ 32.06 < $ 100)

Question 2 :

→ Decision variables:

S_{F1-C1} : unit of ship from factory 1 to customer 1

S_{F1-C2} : " " " " " 1 " " 2.

S_{F1-C3} : " " " " 1 " " 3.

S_{F2-C1} : " " " " 2 " " 1

S_{F2-C2} : " " " " 2 " " 2

S_{F3-C3} : " " " " 2 " " 3.

→ ~~Constraints~~ : Objective function.

$$\min C = 600 S_{F1-C1} + 800 S_{F1-C2} + 700 S_{F1-C3} + 400 S_{F2-C1} + 900 S_{F2-C2} + 600 S_{F2-C3} \text{ (dollars)}$$

→ Constraints.

$S_{F1C1} + S_{F1C2} + S_{F1C3} = 400$ (unit)

$S_{F2C1} + S_{F2C2} + S_{F2C3} = 500$ (units).

$S_{F1C1} + S_{F2C1} = 300$ (units).

$S_{F1C2} + S_{F2C2} = 200$ (units)

$S_{F1C3} + S_{F2C3} = 400$ (units).

$S_{F1C1}, S_{F1C2}, S_{F1C3}, S_{F2C1}, S_{F2C2}, S_{F2C3} \geq 0.$

Question 3.

→ Decision variables:

(3.1) x_1: production level of plates.

x_2: production level of mugs

x_3: production level of steins.

→ Objective function:

Max $P = 3.1x_1 + 4.75x_2 + 4x_3$. ✓ (dollars)

→ Constraints,

$4x_1 + 6x_2 + 3x_3 \leq 2{,}400$ ✓ (minutes)

$8x_1 + 14x_2 + 12x_3 \leq 7{,}200$ ✓ (minutes)

$5x_1 + 4x_2 + 3x_3 \leq 3{,}000$ ✓ (ounces).

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$ ✓

(3.2) Check unit of measurement.

→ objection function:

$$\underset{\text{dollars}}{\text{Max } P} = \underset{\text{dollars}/\cancel{EA} \times \cancel{EA}}{\$3.1\,x_1} ✓ + \underset{\text{dollars}/\cancel{EA} \times \cancel{EA}}{\$4.75\,x_2} ✓ + \underset{\text{dollar}/\cancel{EA} \times \cancel{EA}}{\$4\,x_3}$$

→ Constraints.

$$\underset{\text{minutes}/\cancel{EA} \times \cancel{EA}}{8x_1} ✓ + \underset{\text{minutes}/\cancel{EA} \times \cancel{EA}}{14x_2} ✓ + \underset{\text{minutes}/\cancel{EA} \times \cancel{EA}}{12x_3} \leq \underset{\text{minutes}}{7200}$$

$$\underset{\text{ounces}/\cancel{EA} \times \cancel{EA}}{5x_1} ✓ + \underset{\text{ounces}/\cancel{EA} \times \cancel{EA}}{4x_2} ✓ + \underset{\text{ounces}/\cancel{EA} \times \cancel{EA}}{3x_3} \leq \underset{\text{ounces.}}{3{,}000}$$

(3.3) Report binding constraints and non-zero slack.

→ Binding constraints: Constraint 1 and constraint 2. ✓

Non zero slack : Constraint 3. ✓

complementary slackness

→ Binding because the final resource used (LHS) is equal to available resources which means that the production used up all the resources allocated. (slack = 0). ✓

Non-zero slack because the final resources used (LHS) is less than available resources (RHS) → the production did not used all the allocated resources

Dairy

Q.3.3

The molding process and Finishing process are fully utilized their resources and they has zero slack.

	L.HS		R.HS	slack
clay (ounces)	2,700	$\leq$	3,000	300.

The clay process has not binding because it doesnot use all resources and some resources are left over at the optimal solution.

It has 300 slack value and It means that the clay process will use 2,700 (ounces) although it can use 3,000 ounces

Therefore, the clay process is wastes of resources.

Q 3.4

Non-zero reduced cost

① The non-zero reduced cost of -0.46 for X_2 is the amount will be decreased from the optimal profit when we choose to Produce at least 1 unit of X_2, plates.

② The non-zero reduced cost -0.46 for production mugs X_2 and it indicates how much we have to change the objective function.

The amount of plates per unit would need to be at least 0.46 ($) higher before it would be the optimal to utilize.

+1

Sw Ky

Daisy Tun [IB-6086]

Q. 3.5

Given: the coefficient of plate decision variable increase by $

From Sensitivity Report

Name	Final value	Reduced Cost	Objective Coefficient	Allowable Range *
Production Plates	300	0	3.10	[2.73, 5.33]

* Allowable Range = [3.10 - 0.37, 3.10 + 2.23]
= [2.73, 5.33]

If we increase ($ 2.00) in production plates, the optimal value will be $ 5.1 = $ 3.10 + $ 2.00.

The optimal solution will not change.

2

Q. 3.6

The largest shadow price is 0.28 in the Finishing (minutes used process.

It means that the production of time is increased 1 minutes, the total profit is increased by $0.28.

If the firm deducted from 87,200 available of Finishing (minutes use, the profit will decrease by 0.28. This value of 0.28 is from the range of [7200 - 2400, 7200 + 2400]

[4800, 9600].

Daisy Tun
1B 6086

Q. 3.7

	LHS	R.H.S	A.I	A.D
Molding	2400	2400	200	600

Allowable Range = [2400 - 600 , 2400 + 200]

= [1800, 2600]

But increasing to RHS from 2400 to 3000 , the amount will be out out allowable range. Then our optimal value and optimal soultion will change.
We need to run LP with the new coefficient to get new optimal solution.
The shadow price is no longer valid.
I will advise him if you increase the amount of RHS, we need to run LP with new coefficient.
The shadow price is also change.