INTERNATIONAL UNIVERSITY OF JAPAN

Public Management and Policy Analysis Program
Graduate School of International Relations

DCC5350 (2 Credits)
Public Policy Modeling
Spring 2016

Midterm Exam (100 points)

Instruction: Please write down your student ID and name at the top of your answer. You MUST always show necessary computation and your reasoning as clearly as possible. When handling rational numbers (e.g., 3.1415), use at least three digits below the decimal point.

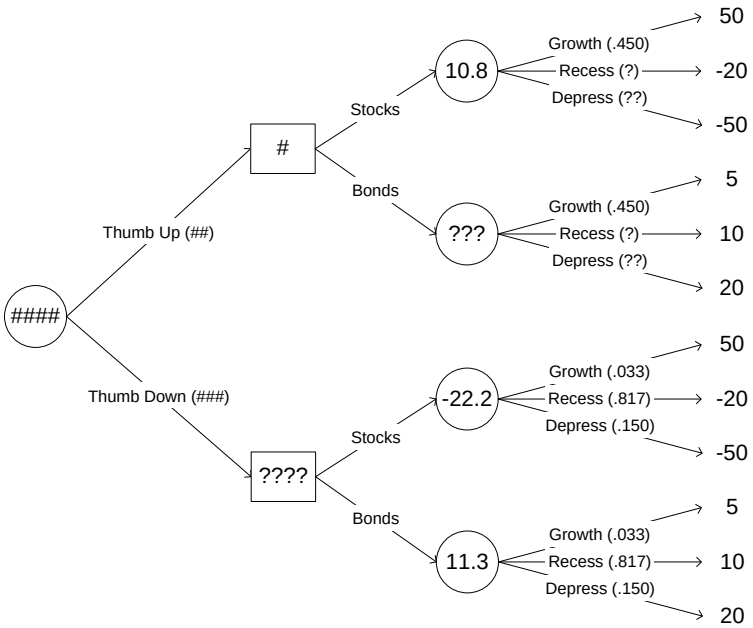
Question 1 (20 points) Suppose your goal is to get A from this Public Policy Modeling at IUJ in 2016.

1.1 (5 points) Classify this problem in terms of level of knowledge, static/dynamic, and strategic/nonstrategic.

1.2 (5 points) Identify at least three decision variables. Specify the unit of measurement.

1.3 (5 points) Identify at least three important environment variables. Do not say, for example, Obama's preference (favorable/unfavorable) for immigration policy that is rarely related to your getting A. Specify the unit of measurement.

1.4 (5 points) Identify at least three criterion variables. Specify the unit of measurement.



Question 2 (30 points) Read Question 9.27 on page 375 and look at the decision tree under imperfect (sample) information above. Some payoffs were changed and prior probabilities are P(Growth)=.2, P(Recession)=.7, and P(Depression)=.1. Economy guru will thumb up or down to indicate his forecast and conditional probabilities are P(Up|Growth)=.9, P(Up|Recession)=.3, and P(Up|Depression)=.1.

2.1 (3 points) Construct a decision table (payoff table) with prior probabilities incorporated.

2.2 (3 points) Make a decision using EMV.
2.3 (3 points) Construct the regret table.
2.4 (3 points) Make a decision using EOL.
2.5 (4 points) Calculate the expected value of perfect information. Draw a decision tree under perfect information.
2.6 (7 points) Fill the blanks of ?, ??, ???, and ???? in the decision tree above.
2.7 (4 points) Fill the blanks of #, ##, ###, and #### in the decision tree above.
2.8 (3 points) If the guru asks 1 (price) for his forecasting, would you like to buy it? Why?

Question 3 (30 points) Look at Q5.8 on page 184.
3.1 (5 points) Report the optimal solution and optimal value (of the objective function). Show your calculation.
3.2 (5 points) Interpret the meaning of the allowable increase and decrease of the third coefficient. You need to mention at least optimal solution and optimal value.
3.3 (5 points) Interpret the meaning of a non-zero reduced cost substantively.
3.4 (5 points) Fill the first (Final Value) and third (Constraint R.H.Side) blanks of constraint 1 (Milk) in the second part of the Excel output. Show your calculation and/or reasoning. Ignore allowable increase and decrease.
3.5 (5 points) Report slack/surplus of constraint 1. Show your calculation. Is it slack or surplus? What is the shadow price of constraint1? How did you know that?
3.6 (5 points) Explain the meaning of the largest shadow price substantively. Don't forget to add the allowable range in your explanation.

Question 4 (20 points) Remember the group project 1. Here you may use your group's answer or your own answer (if you don't agree on the group answer). The following is the sensitivity report of Q2.7.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B15	x1	12	0	3.3	1E+30	2.15
C15	x2	4	0	**2.3**	4.3	15.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B17	Constraitn 1 x1	4	0	3	1	1E+30
B18	Constraitn 2 x1	44	0	18	26	1E+30
B19	Constraitn 3 x1	4	-0.716666667	4	36	6
B20	x1	20	2.583333333	20	1E+30	6

4.1 (3 points) Show how 2.3 under `Objective Coefficient` was calculated.
4.2 (5 points) Check the unit of measurement of the objective function and second constraint.
4.3 (3 points) Explain why reduced costs above are all zero.
4.4 (5 points) Given the information above (don't try to run Excel Solver), what would you suggest for constraint 3 to maximize the benefit? You must mention what you want to change and its consequence (result).
4.5 (4 points) Among certainty, divisibility, proportionality, additivity, and homogeneity, which assumption do you think is most problematic in this LP formulation? And why?

End of the midterm exam.

IBS029
LOUIE

100 - 4 = 96

+ 4 extra credits.

Q1.

1.1 uncertain, dynamic, strategic. why? -2

1.2 a.) amount of study time, hrs.
b.) score in HW, projects, midterms : points
c.) attendance in class ✓ -2

1.3 a.) no. of books available in the library for borrowing, units/pc.
b.) available time for consultation w/ TA / Dr. Park, hrs.
c.) grading policy set by IUJ.

1.4) a. exam result.
b. group project score
c. hw score
} should be at least 96% to get A.

Louie, I love your handwriting and careful reasoning :D

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LOUIE

Q2.

2.1 Payoff Table (in % of return-of-investment)

Payoff / Alternative	State of Nature		
	Growth	Recession	Depression
Invest in Stocks	50	-20	-50
Invest in Bonds	5	10	20
Prior Probability	0.2	0.7	0.1

2.2 Decision w/ EMV

$EMV = \sum (\text{prior probability})(\text{payoff})$

$EMV, \text{stocks} = (0.2)(50) + (0.7)(-20) + (0.1)(-50)$

$= -9.00\%$

$EMV, \text{bonds} = (0.2)(5) + (0.7)(10) + (0.1)(20)$

$= 10\%$

$EMV, \text{bonds} > EMV, \text{stocks}$

$10\% > -9.00\%$

therefore, invest in bonds bec. it gives better return-of-investment.

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LOUIE

2.3 Regret Table

Regret / Alternative	State of Nature: Growth	Recession	Depression
Invest in Stock	0	30	70
Invest in Bonds	45	0	0
Prior Probability	0.2	0.7	0.1

Regret (stocks | growth) = 50 - 50 = 0
Regret (stocks | Recession) = 20 - (-10) = 30
Regret (stocks | Depression) = 30 - (-20) = 70
Regret (bonds | growth) = 50 - 5 = 45
" (bonds | Recession) = 10 - 10 = 0
" (bonds | Depression) 20 - 20 = 0.

2.4) Decision w/ EOL : $EOL = \Sigma(\text{prior probability})(\text{regret})$

$EOL, \text{stocks} = (0.2)(0) + (0.7)(30) + (0.1)(70)$
$= 28\%$

$EOL, \text{bonds} = (0.2)(45) + (0.7)(0) + (0.1)(0)$
$= 9\%$

$EOL, \text{bonds} < EOL \text{ stocks}$
$9\% < 28\%$

So, invest in bonds bec. it provides lower risk / smaller regret.

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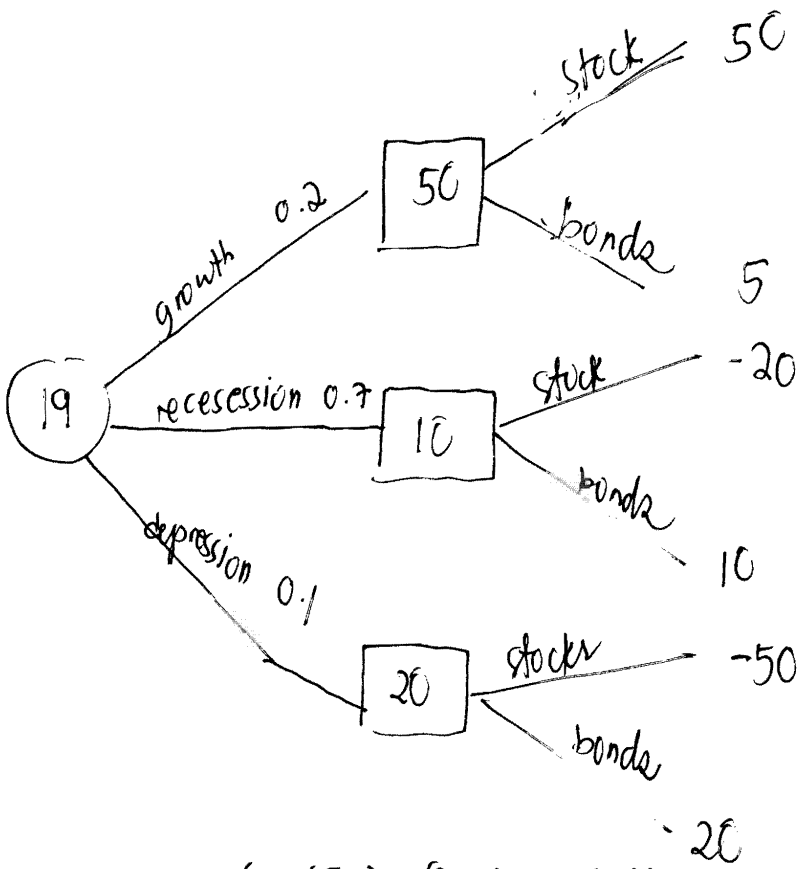
2.5

Payoff



$EP, \text{perfect info} = (0.2)(50) + (0.7)(10) + (0.1)(20)$

$= 19$

$EVPI = (EP, \text{perfect info}) - (EMV)$

$= 19 - 10$

$EVPI = 9$

any info that costs beyond this value is not worthwhile to buy.

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OUIE

Let Growth = G
Recession = R
Depression = D

1.6) given:

$P(G) = 0.2$
$P(R) = 0.7$
$P(D) = 0.1$
$P(UP|G) = 0.9$
$P(UP|R) = 0.3$
$P(UP|D) = 0.1$

by collective exhaustiveness:

$P(Down|G) = 0.1$
$P(Down|R) = 0.7$
$P(Down|D) = 0.9$

$P(Up \cap G) = P(UP|G) * P(G) = 0.9 \times 0.2 = 0.18$ ✓
$P(Up \cap R) = P(UP|R) * (P(R)) = 0.3 \times 0.7 = 0.21$ ✓
$P(Up \cap D) = P(UP|D) = P(D) = 0.1 \times 0.1 = 0.01$ ✓
$P(Down \cap G) = 0.1 \times 0.2 = 0.02$
$P(Down \cap R) = 0.7 \times 0.7 = 0.49$
$P(Down \cap D) = 0.9 \times 0.1 = 0.09$

not needed at all.

probabilities Table

	G	R	D	
Up	0.18	0.21	0.01	0.4
Down	0.02	0.49	0.09	0.6
	0.20	0.70	0.10	1

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Lonie

2.6) ?: $P(R \mid Up) = \frac{P(R \cap Up)}{P(up)} = \frac{0.21}{0.4} =$ **0.525** ✓

?? : $P(D \mid Up) = \frac{P(D \cap Up)}{P(up)} = \frac{0.01}{0.4} =$ **0.025** ✓

??? : $EP_{,(bonds \mid UP)} = \frac{(0.450)(5) + (0.525)(10) + (0.025)(20)}{}$

$= 8.$ // ✓

???? : $EP_{,down} = 11.3$ // (bec ✓

EP (bonds | down) > EP (stocks | down)
11.3 > -22.2

Good D

2.7 given UP : EP stocks > EP bonds
10.8 > 8 ✓

choose # = 16.8

= $P(Up) = 0.4$ ✓ (from the probability table)
(= $P(Up \cap G) + P(Up \cap R) + P(Up \cap D) = 0.18 + 0.21 + 0.01$

= $P(Down) = 0.60$ ✓ (by collective exhaustiveness, $P(Up) + P(down) = 1$

$P(down) = 1 - P(Up)$
$= 1 - 0.4$
$= 0.6$ //

= $(0.4)(16.8) + (0.6)(11.3)$
$=$ **11.1** ✓

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QUIZ

2.8) EVSI = EVII - EMV
EVSI = 11.1 - 10 ✓
EVSI = 1.1 ✓

bec EVSI > cost of info
1.1 > 1,

info is worthwhile to buy. Get guru's advice!

Perfect answer!

Q3.

3.1 optimal sol'n is to produce:

x_1 = 0 gallons of chocolate ice cream
x_2 = 300 gallons of vanilla " "
x_3 = 75 " " banana " ".

max profit = $x_1 + 0.90x_2 + 0.95x_3$
= $0 + (0.90)(300) + 0.95(75)$
Π = \$341.25 ✓

3.2) the unit profit for banana (x_3) can range from [0.90, 0.9714] w/o affecting the optimal solutions of $x_1 = 0$, $x_2 = 300$ and $x_3 = 75$. But obviously the value of the profit will change. ✓

LOUIE
1B5029

3.3) The non-zero reduced cost of -0.0375 for X_1 is the amt. that will be decreased from the profit if we choose to produce at least 1 gallon of X_1 chocolate ice cream.

3.4) milk
Final Value: 180 ✓ $= (0.45)(0) + (300)(0.5) + 75(0.4)$
Constraint RHS: 200 ✓ → 200 is the maximum amount of milk available to produce the 3 kinds of ice cream.

3.5) Slack of milk

$|RHS - LHS| = |200 - 180| = 20$ ✓

it is a slack bec it involves the sign of $\leq$. ✓
It means that some milk are underutilized.

Shadow price of milk: ✓

0 → bec. it is a nonbinding constraint.

3.6) The shadow price of 1.875 for sugar is the amount that will be added/subtracted to the maximized profit for every 1 gallon increase/decrease in the available sugar. This price is valid from [120, 160]. Meaning, if there is increase in the sugar resource from 150 to 160, the profit will increase by $18.75 (= 1.875 \times 10$].

Louie
IB5029

4.1 Π = (total profit) − (total cost)

= [0.5SR + 1.5LD + 0.2PD) − (1TV + 2PM)

= [0.5(0TV + 1PM) + 1.5(3TV + 2PM) + 0.2(−TV + 4PM)] − [1TV + 2PM]

= 0.5PM + 4.5TV + 3PM − 0.2TV + 0.8PM − TV − 2PM

= 3.3TV + 2.3PM ✓

4.2) Constraint 2:

3TV + 2PM ≥ 18

$(3\ \%/TV)(1\ TV) + (2\ \%/PM)(1\ PM) \geq 18\ \%$ ✓

% ≥ % ok.

Objective function:

Π = 3.3TV + 2.3PM

$\$ = (3.3\ \$/TV\ units)(1\ TV\ units) + (2.3\ \$/PM\ units)(1\ PM\ units)$

$\$ = \$$ ok.

4.3) Zero reduced cost is observed since we have non-zero final values for TV and PM. ✓ (4 and 3, respectively).

4.4) relax constraint to 0 (instead of 4), and this will add $(4 \times 0.72) = \$2.88M$ in total profit. Great! + any additional

4.5) proportionality. In this LP problem, we assumed that advertising units will bring in the same increase in sales. But, in reality there is such a thing as saturation: there is marginal decrease in sales for every ad units done. + yes!

Public Policy Modeling 2016
Midterm Exam
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Question 1

1.1
- uncertainty because I don't know probabilities of outcomes
- dynamic because time plays a role during the term.
- strategic because students and instructor interact strategically.

1.2
- The number of hours for studying per day.
- The number of hours for group discussion per week
- Whether or not I attend class per week.
- The number of questions asked in class per week

1.3
- IUJ Regulations for cheating
- Course policies described in the syllabus.
- Long winter season that influences my studying

1.4
- Score I earned in the midterm and final exams
- Score I earned in homeworks
- The number of extra credits received
- Score I earned in quiz

Question 2

2.1

	Growth	Recession	Depression
Stocks	50	-20	-50
Bonds	5	10	20
Probability	.2	.7	.1

2.2 EMV of stocks: $50*(.2)+(-20)*(.7)+(-50)*(.1) = -9$

EMV of bonds: $5*(.2)+(10)*(.7)+(20)*(.1) = 10$

So invest on bonds. $(-9 < 10)$

2.3

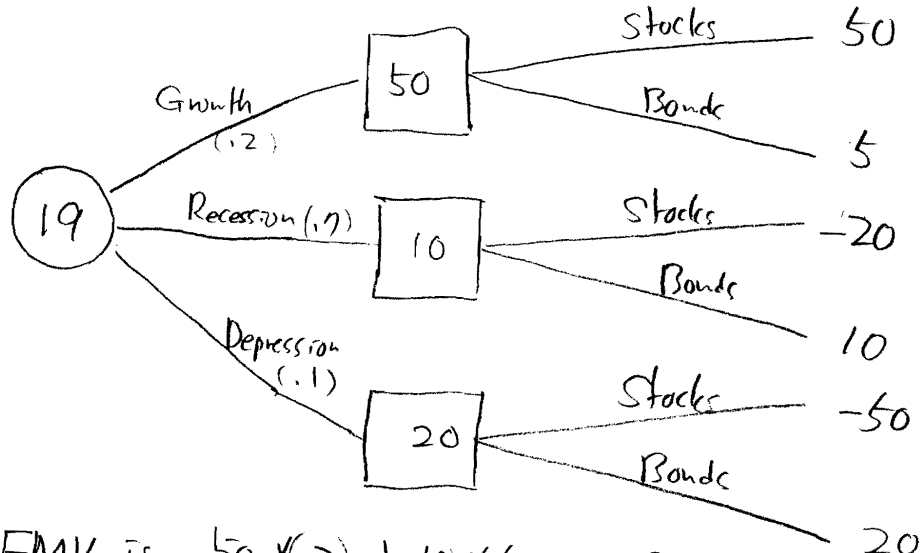
	Growth	Recession	Depression
Stocks	$0 = 50-50$	$30 = 10-(-20)$	$70 = 20-(-50)$
Bonds	$45 = 50-5$	$0 = 10-10$	$0 = 20-20$
Probability	.2	.7	.1

2.4 EOL of stocks: $0*.2+30*.7+70*.1 = 28$

EOL of bonds: $45*.2+0*.7+0*.1 = 9$

So invest on bonds because $28 > 9$.

2.5



EMV is $50*(.2)+10*(.7)+20*(.1) = 19$.

EVPI $= 19-10 = 9$ (from 2.2)

2.6 $P(\text{Recession} \mid \text{Up}) = \dfrac{P(\text{Recession} \cap \text{Up})}{\text{Marginal Probability of Up}}$

MP of Up $= P(\text{Recession} \cap \text{Up}) + P(\text{Growth} \cap \text{Up}) + P(\text{Depression} \cap \text{Up}) = P(\text{Up})$

$P(R \cap \text{Up}) = P(\text{Up} \mid \text{Recession}) * P(\text{Recession})$

$= .3 * .7 = .21$

$P(G \text{ and } Up) = P(Up \mid Growth) * P(Growth)$

$= .9 * .2$

$= .18$

$P(D \text{ and } Up) = P(Up \mid Depression) * P(Depression)$

$= (.1) * (.1) = .01$

$P(Up) = .21 + .18 + .01 = .40$

$P(Recession \mid Up) = \frac{.21}{.40} = .525$ ..... ?

$P(Depression \mid Up) = \frac{.01}{.40} = .025$ ..... ??

EMV of bonds given $Up = 5*(.45) + 10*(.525) + 20*(.025)$

$= 2.25 + 5.25 + .5$

$= 8$ ..... ???

???? = 11.3 because $11.3 > -22.2$

2.7 # is 10.8 because $10.8 > 8$

is .40 = $P(Up)$

is .60 = $1 - P(up) = 1 - .4 = P(Down)$

(EV under imperfect information)

$= 10.8 * (.4) + 11.3 * (.6)$

$= 4.32 + 6.78$

$= 11.1$

2.8 EVII = $11.1 - 10 = 1.1$

I will buy the information since $1.1 > 1$.

Question 3

3.1 Chocolet = 0 gallons
Vanilla = 300 gallons
Banana = 75 gallons

Optimal value is $345.25
$= \$1.*0 + \$.90 * 300 + \$95 * 75.$

3.2 As long as current .95 changes from .90 (=.95 − .05) to .9714 (=.95 + .0214), the current optimal solution remains unchanged. but the optimal value will change.

3.3 When chocolete is produced at least one gallon, company's profit will decrease by $.0315 per gallon.

3.4 Final value is $180 = .45*0 + .5*300 + .4*75$.
RHS is 200 that is given

3.5 Slack is 20 = 200 − 180 because the constraint has ≤. Shadow price is 0 because slack is not zero.

3.6 Whenever the company increase RHS of the second constraint by 1 unit, the profit will increase by $1.875 as long as RHS stays within from 120 (= 150 − 30) to 160 (150 + 10).

Question 4

4.1 Profit = Revenue − Cost
$= (.5)x_2 + 1.5)(3x_1 + 2x_2) + (.2)(-x_1 + 4x_2)$
$- (x_1 + 2x_2)$

$= .5x_2 + 4.5x_1 + 3x_2 - .2x_1 + .8x_2 - x_1 - 2x_2$

$= (4.5 - .2 - 1)x_1 + (.5 + 3 + .8 - 2)x_2$

$= 3.3x_1 + 2.3x_2$

2.3 is calculated in this manner.

4.2 Objective function

$\$Millions = (\$Millions/EA) * (EA) + (\$Millions/EA) * EA$

$= \$Millions$ ... O.K.

Constraint 1

$\%\ increase = (\%\ increase/EA) * (EA) + (\%\ increase/EA) * EA$

$= \%\ increase$ ... O.K.

4.3 Optimal solution is not zero.

4.4 Since shadow price is negative, the company needs to decrease RHS of constraint 3. RHS can be reduced to -2 (= 4-6), but RHS should be zero or positive number. Therefore, company have to reduce RHS to zero in order to get $2.8667 million increase in profit. (= .71667 * 4)

4.5 Divisibility is problematic because advertisement should be integer. Homogeneity may be problematic if each advertisement has a different impact. Also proportionality may not be satisfied if we expect a diminishing effect of advertisement; coefficients in constraints will decrease as the number of advertisement increases.