INTERNATIONAL UNIVERSITY OF JAPAN

Graduate School of International Relations

Math Training Sessions (Fall 2012)
by Hun Myoung Park

Solving Equations

1. Reading Mathematical Expressions

- a=b → *a* equals *b*"
- a≈b → *a* is approximately equal to *b*"
- a×b → "*a* times *b*" "*a* multiplied by *b*"
- a÷b → "*a* divided by *b*"
- a/b → "*a* over *b*"
- 1/a → "one *a*th" (e.g., 1/3 → "one third")
- a>b → "*a* greater than *b*"
- a≤b → "*a* less than or equal to *b*"

- $-\infty$ → "negative infinity"
- |a| → "absolute value of *a*"
- a! = $a\times(a-1)\times(a-2)\ldots\times2\times1$ → "*a* factorial"
- $\sum_a^b x_i$ → "Summation from *i* equals *a* to *b* of x_i" or "Sum of x_i for *i* from *a* to *b*"

- a ∈ B → "*a* is an element of set *B*"
- A ⊃ B → "*A* is a subset of *B*" or "*A* is contained in *B*"

- a^2 → "*a* squared"
- a^b → "*a* to the *b*th power"
- a^{-1} → "*a* inverse"
- $\sqrt{a}$ → "square root of *a*"
- $\sqrt[b]{a}$ → "*b*th root of *a*"

- e^a → "exponential of *a*" or "*e* to the *a*"
- *nl*(a)=log(a)= $\log_e^a$ → "natural log(arithm) of *a*"
- $\log_b a$ → "log(arithm) in base *b* of *a*" or "log *a* base *b*"

- $\lim_{x\to a} f(x)$ → "limit as *x* goes to *a* of *f(x)*" or "limit of *f(x)* as *x* goes to *a*"
- $\frac{\partial f}{\partial x}$ → "partial derivative of *f* with respect to *x*"
- $f'(x)$ → "*f* prime of *x*" or "the (first) derivative of *f* with respect to *x*"

2. Sets

- Element: x ∈ A; x ∉ A
- Subset: A ⊃ B or B ⊂ A
- Null set (empty set): ϕ or {}

- Union of two sets including all elements belonging to A or B: $A \cup B$
- Intersection of two sets including all elements belonging to A and B: $A \cap B$
- Complement of A: $\tilde{A} = \{x \mid x \in U \text{ and } x \notin A\}$, where U is a universal set
- Commutative law: $A \cup B = B \cup A$ $A \cap B = B \cap A$
- Associative law: $A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive law: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3. Solving Equations

3.1 Addition and subtraction properties of equality. If two expressions are equal to each other and you add or subtract the exact same thing to both sides, the two sides will remain equal.

If $a = b$, then $a + c = b + c$
If $a = b$, then $a - c = b - c$

3.2 Multiplication and division properties of equality. If two expressions are equal to each other and you multiply of divide (except for 0) the exact same constant to both sides, the two sides will remain equal.

If $a = b$, then $ac = bc$
If $a = b$, the $a/c = b/c$ where $c \neq 0$

3.3 Elimination and substitution for simultaneous equations. In order solve a simultaneous linear equation (a set of linear equations), you may successively eliminate variables and equations through substitution or subtraction. For example,

$$\begin{cases} x + 3y = 4 \\ -x + 2y = 6 \end{cases}$$

Substitution:

a) From the first equation, $x = 4 - 3y$.
b) Plug in into the second equation. $-(4 - 3y) + 2y = 6$; $-4 + 3y + 2y = 6$; $5y = 10$; $y = 2$.
c) Then plug $y = 2$ back in to the first equation.
d) $x + 3(2) = 4$; $x = 4 - 6$; $x = -2$

Subtraction (addition) to eliminate one variable:

a) Sum of two equations side by side
b) $(x + 3y) + (-x + 2y) = 4 + 6$; $5y = 10$; $y = 2$
c) Then plug $y = 2$ back in to the first equation.
d) $x + 3(2) = 4$; $x = 4 - 6$; $x = -2$

In Maple,

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> solve({x+3*y=4, -x+2*y=6},{x,y});
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{x = -2, y = 2}

3.4. Graphical solutions. The solution is the place where lines intersect.

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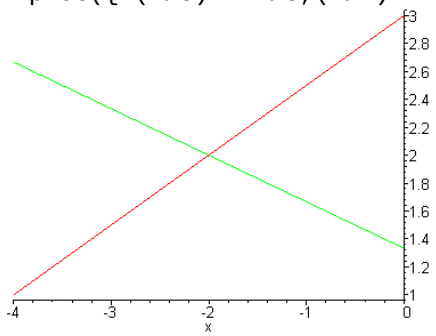
> plot({-(1/3)*x+4/3,(1/2)*x+3}, x=-4..0);
```



3.5 Factoring for higher order equations. If factoring of a higher older equation is available, use "If *ab*=0, the *a*=0 or *b*=0."

- $(x+y)(x+y)=x^2+2xy+y^2$
- $(x+y)(x-y)=x^2-y^2$
- $(x-y)(x-y)=x^2-2xy+y^2$
- $(ax+by)(cx+dy)=acx^2+(ad+bc)xy+bdy^2$

Example,

$-2x^2+7x-3=-(2x-1)(x-3)$

$(2x-1)=0\,;\ x=1/2$

$(x-3)=0\,;\ x=3$

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> plot({-2*x^2+7*x-3}, x=0..4);
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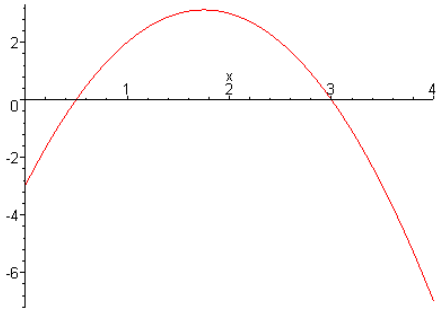

3.6 Quadratic formula for the second order equations

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

3.7 Numerical solutions for functions. Plug in a series of numbers and evaluate values of *y*.