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Math Training Sessions (Fall 2012)
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1. What Is a Matrix?

A *matrix* is a rectangular array of numbers. It is a collection of numbers in a rectangular form. You may read an element of a matrix as, $a_{row,column}$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nk} \end{bmatrix}$$

Why do you need to study matrices? Matrices provide easy ways of expressing and solving linear equations. Its simplicity and succinctness are very useful and even essential in econometrics.

2. Examples

Suppose you have a simultaneous equation set with two linear equations.

$$\begin{cases} 2x + 3y = 16 \\ -x + 2y = 6 \end{cases}$$

You may solve this equation system by substitution. Plug the second equation into the first.

$$\begin{aligned} -x + 2y &= 6 \\ x &= -6 + 2y \end{aligned}$$

$$\begin{aligned} 2(-6 + 2y) + 3y &= 16 \\ -12 + 4y + 3y &= 16 \\ 7y &= 28 \\ y &= 4 \end{aligned}$$

$$\begin{aligned} -x + 2(4) &= 6 \\ -x + 8 &= 6 \\ x &= 2 \end{aligned}$$

Now consider the following expression using matrices, which is the identical to the equation system above.

$$\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

Let us call $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ a matrix A, $\begin{bmatrix} x \\ y \end{bmatrix}$ a matrix X, and $\begin{bmatrix} 16 \\ 6 \end{bmatrix}$ a matrix B. Then the expression will be simpler: AX=B. And the solution is quite simple: $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 16 \\ 6 \end{bmatrix} = \frac{1}{2\cdot 2 - (-1)3} \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 16 \\ 6 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 2\cdot 16 - 3\cdot 6 \\ 1\cdot 16 + 2\cdot 6 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 14 \\ 28 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

The classical linear regression model (CLRM) or ordinary least square (OLS) is presented in the following matrix form.

y=Xb

where y is a column vector of the dependent variable, X is a matrix of 1 and independent variables, and b is the column vector of parameter estimates. The parameter estimates for the regression equation is calculated as,

$$b = (X'X)^{-1}X'y$$

In matrix algebra, the calculation is a piece of cake when using matrix-based software packages like MATLAB, SAS/IML, R, and Stata/Mata.

3. Diversity of Matrices

There are various types of matrices each of which has its own purpose.

- A *row vector* is a matrix with only one row, while a *column vector* is a matrix with one column.
- A *square matrix* is a matrix that has the same number of row and column.
- An *identity matrix* is a matrix with ones on its diagonal and zeros on all other elements.
- A *zero matrix* is a matrix whose elements are all zero.
- A *partitioned matrix* is a matrix whose elements are grouped into submatrices.

4. Matrix Operations

- $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

5. Inverse Matrices

- Available only for square matrices.
- When $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- Transpose: X'= $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ (X')'=X (XY)'=Y'X'
- Determinant: |X|=ad - bc
- *adj* X= $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- Inverse matrix: $X^{-1} = \frac{1}{|X|}$ adj X;
- $(X^{-1})^{-1} = X$ $(XY)^{-1} = Y^{-1}X^{-1}$