K300 (4392) Statistical Techniques (Fall 2007)

Lecture Note: Hypothesis Testing 2

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1. Before and After the Central Limit Theorem

Central Limit Theorem: $\bar{x} \sim N(\mu, \sigma^2)$, if N is large

Random available x		Sample mean $\bar{x}$
$x_1, x_2, x_3, \dots x_n$	Data	$\bar{x}_1$, ($\bar{x}_2$, $\bar{x}_3$, …)
May or may not be normally distributed	Normality	Normally distributed if N is large (CLM)
$\bar{x} = \frac{\sum x_i}{n}$	Mean	
$Var(x) = \frac{\sum (x_i - \mu)^2}{n} = \sigma^2$ $Var(x) = \frac{\sum (x_i - \bar{x})^2}{n-1} = s^2$	Variance	$Var(\bar{x}) = Var\left(\frac{\sum x_i}{n}\right) = \left(\frac{1}{n}\right)^2 Var\left(\sum x_i\right)$ $= \left(\frac{1}{n}\right)^2 \left(\sigma^2 + \sigma^2 + \sigma^2 \dots\right) = \frac{\sigma^2}{n}$
$SD(x) = \sqrt{\sigma^2} = \sigma$ (population) $SD(x) = \sqrt{s^2} = s$ (sample)	Std. Dev.	$SD(\bar{x}) = s_{\bar{x}} = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$ (standard error)
$z_x = \frac{x - \mu}{\sigma}$, $z_x = \frac{x - \bar{x}}{s}$ (sample)	Z-test	$z_{\bar{x}} = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
	CI (Z)	$\bar{x} - z_{\alpha/2} \times \sigma_{\bar{x}} \le \mu \le \bar{x} + z_{\alpha/2} \times \sigma_{\bar{x}}$
	T-test	$t_{\bar{x}} = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{\bar{x} - \mu}{s / \sqrt{n}}$ (when σ is unknown)
	CI (T)	$\bar{x} - t_{\alpha/2} \times s_{\bar{x}} \le \mu \le \bar{x} + t_{\alpha/2} \times s_{\bar{x}}$

* The standard error is the standard deviation of sample mean.

2. Example 8-5, Page on 404

σ=3251, μ=24672, N=35, sample mean=25226.
Since σ is known, we can use the normal distribution (z-test)

2.1 Test statistic approach:

i. H_0: μ=24672, H_a: μ≠24672 This is a two-tailed test. So, you have to consider both extremes.
ii. *α=.01 (rejection region), critical values are ±2.58*
iii. $z_{\bar{x}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{25226 - 24672}{3251 / \sqrt{35}} = 1.01$
iv. *Since TS (1.01) < CV (2.58),* do not reject the null hypothesis at the .01 level.
v. The average cost of stroke rehabilitation is $24,672.

2.2 P-value approach:

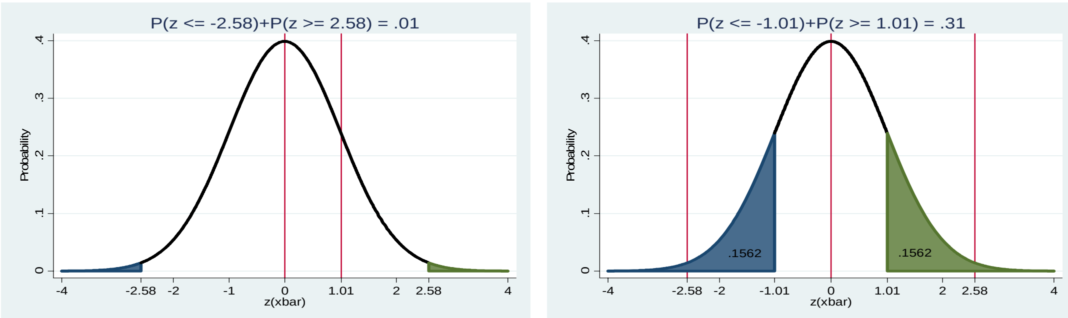
i. H_0: μ=24672, H_a: μ≠24672 This is a two-tailed test. So, you have to consider both extremes.

ii. α=.01 (rejection region)

iii. $z_{\bar{x}} = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{25226-24672}{3251/\sqrt{35}} = 1.01$, *p-value is .3125 since* $P(z_{\bar{x}} \geq 1.01) = .1562$

iv. *Since p-value (.3125) > α (.01),* do not reject the null hypothesis at the .01 significance level.

v. The average cost of stroke rehabilitation is $24,672.



2.3 Confidence Interval approach:

i. H_0: μ=24672, H_a: μ≠24672 This is a two-tailed test. So, you have to consider both extremes.

ii. *α=.01 (rejection region),* 99 percent confidence level, *critical values are ±2.58*

iii. $\bar{x} \pm z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}} = 25226 \pm 2.58 \times \dfrac{3251}{\sqrt{35}} = 25226 \pm 1418$, *[23808, 26644]*

iv. *The hypothesized mean 24672 exists between 23808 and 26644.* Therefore, we do not reject the null hypothesis at the .01 level.

v. The average cost of stroke rehabilitation is $24,672.

3. Example 8-6, Page on 407

s=2, μ=24, N=36, sample mean=24.7.

Since σ is not known but N≥30, we may use the normal distribution (z-test). But I prefer t-test to z-test as long as σ is unknown because I am a conservative person.

3.1 Test statistic approach:

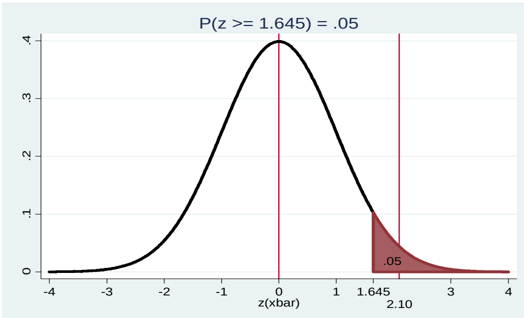
i. H_0: μ≤24672, H_a: μ>24672 This is a right one-tailed test. So, you have to consider the positive extreme.

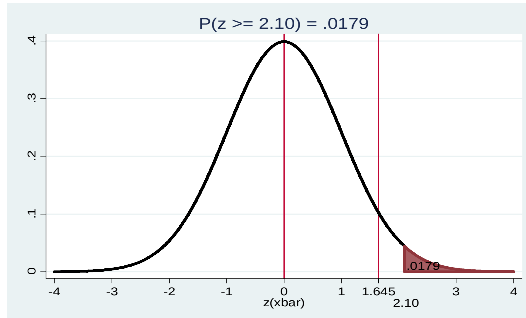
ii. α=.05 (rejection region), the critical value is +1.645

iii. $z_{\bar{x}} = \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{24.7-24}{2/\sqrt{36}} = 2.10 \sim N(0,1)$

iv. *Since TS (2.10) ≥ CV (2.58),* reject the null hypothesis at the .05 level.

v. The average age of lifeguards in Ocean City is greater than 24.





3.2 P-value approach:

i. H_0: μ≤24672, H_a: μ>24672 This is a right one-tailed test. So, you have to consider the positive extreme.

ii. *α=.05 (rejection region)*

iii. $z_{\bar{x}} = \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{24.7-24}{2/\sqrt{36}} = 2.10 \sim N(0,1)$, *p-value* $P(z_{\bar{x}} \geq 2.10) = .0179$

iv. *Since p-value (.0179) < α (.05),* reject the null hypothesis at the .05 level.

v. The average age of lifeguards in Ocean City is greater than 24.

3.3 Confidence interval approach:

Since this is a one-tailed test, it is not relevant to construct the confidence interval.

Now, let us see what will happen when we conduct the t-test?

3.4 Test statistic approach:

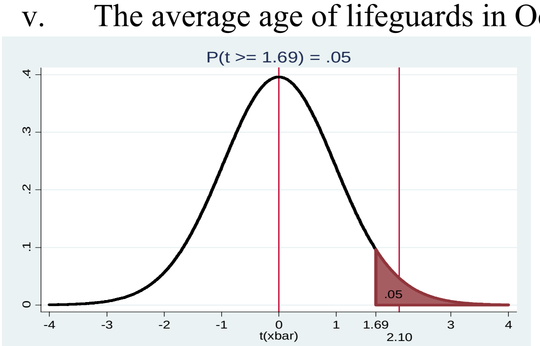
i. H_0: μ≤24672, H_a: μ>24672 This is a right one-tailed test. So, you have to consider the positive extreme.

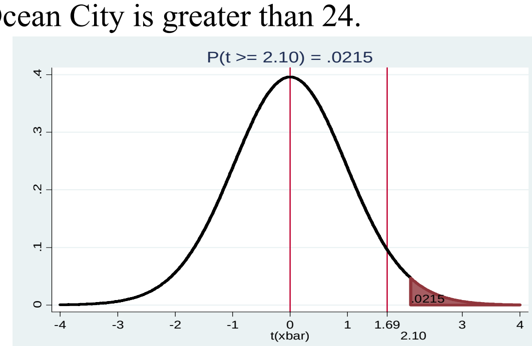
ii. *α=.05 (rejection region), the critical value is +1.69 slightly large than 1.645. The degrees of freedom is 35=36-1.*

iii. $t_{\bar{x}} = \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{24.7-24}{2/\sqrt{36}} = 2.10 \sim t(n-1)$ The computation remains unchanged.

iv. *Since TS (2.10) ≥ CV (1.69),* reject the null hypothesis at the .05 level.

v. The average age of lifeguards in Ocean City is greater than 24.





3.5 P-value approach:

i. H_0: μ≤24672, H_a: μ>24672 This is a right one-tailed test. So, you have to consider the positive extreme.

ii. *α=.05 (rejection region)*

iii. $t_{\bar{x}} = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{24.7-24}{2/\sqrt{36}} = 2.10 \sim t(n-1)$, *p-value* $P(t_{\bar{x}} \geq 2.10) = .0215$

iv. *Since p-value (.0215) < α (.05),* reject the null hypothesis at the .05 level.

v. The average age of lifeguards in Ocean City is greater than 24.

Note that the t distribution depends on the degrees of freedom (n-1). When N is small (say, less than 100), the critical value in the t distribution is larger than that in the standard normal distribution. Due to the larger critical value, the t-test is less likely than the z-test to reject the null hypothesis. That is the reason why I put a label of "conservative."

4. Example 8-12, Page on 417

s=400, μ=24,000 N=10, sample mean=23450.

Since σ is not known and N<30, we need to conduct the t-test.

4.1 Test statistic approach:

i. H_0: μ=24000, H_a: μ≠24000 This is a two-tailed test. So, you have to consider both extremes.

ii. *α=.05 (rejection region), the critical values are ±2.262, The degrees of freedom is 9=10-1.*

iii. $t_{\bar{x}} = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{23450-24000}{400/\sqrt{10}} = -4.35 \sim t(9)$

iv. *Since TS (|-4.35|) ≥ CV (2.262),* reject the null hypothesis at the .05 level. The test statistic is farther away from the mean 0 than the critical value. It is not likely to get such a sample mean if the null hypothesis is true. So, the hypothesized value 24000 is doubtful.

v. Therefore, the average starting salary for nurses is not $24,000.

4.2 P-value approach:

i. H_0: μ=24000, H_a: μ≠24000 This is a two-tailed test. So, you have to consider both extremes.

ii. *α=.05 (rejection region)*

iii. $t_{\bar{x}} = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{23450-24000}{400/\sqrt{10}} = -4.35 \sim t(9)$, *p-value* $P(t_{\bar{x}} \leq -4.35) = .0009$

iv. *Since p-value (.0009) < α (.05),* reject the null hypothesis at the .05 level. It is extremely less risky to reject the null hypothesis in favor of the alternative hypothesis. You just take only .09 percent of risk when you reject the null hypothesis. Why not reject the null hypothesis!

v. Therefore, the average starting salary for nurses is not $24,000.

4.3 Confidence interval approach:

i. H_0: μ=24000, H_a: μ≠24000 This is a two-tailed test. So, you have to consider both extremes.

ii. *α=.05 (rejection region), 95 percent confidence interval, the critical values are ±2.262 with 9 degrees of freedom.*

iii. $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 23450 \pm 2.262 \times \frac{400}{\sqrt{10}} = 23450 \pm 286$, [23164, 23736]

iv. *The hypothesized mean 24000 exists out of the confidence interval between 23164 and 23736.* Therefore, we reject the null hypothesis at the .05 level.

v. Therefore, the average starting salary for nurses is not $24,000.

5. Question 20, Page on 263

Sample mean: 33.5, sample standard deviation: 27.6777, N=10. This is a confidence interval approach.

1) H_0: μ=unknown

2) *α=.02 (rejection region), 98 percent confidence interval,* and the degrees of freedom are 9=10-1. The critical values of the two-tailed test at the .02 significance level are ± 2.821. Check the t distribution table.

3) $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 33.5 \pm 2.821 \times \frac{27.6777}{\sqrt{10}} = 33.5 \pm 24.6907$, *[8.8093,58.1907]*

4) No hypothesized population mean is provided.

5) N/A

6. Comparing Proportions of Binary Variable

Expected value of a binomial distribution: *np*

Variance of a binomial distribution: *np(1-p)=npq*

$$z = \frac{y-\mu}{\sigma} = \frac{n\hat{p}-np}{\sqrt{np(1-p)}} = \frac{\frac{n\hat{p}-np}{n}}{\frac{\sqrt{np(1-p)}}{n}} = \frac{\hat{p}-p}{\sqrt{p(1-p)/n}} = \frac{\hat{p}-p}{\sqrt{pq/n}}$$

where $\hat{p}$ is the proportion of success in a sample, p is a hypothesized proportion.

The *(1-α)* percent confidence interval of p is based on the sample proportion $\hat{p}$.

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

7. Example 8-17, Page on 426-427

p=.15, q=1-.15=.85, N=200, x=38, $\hat{p} = 38/200 = .19$

7.1 Test statistic approach:

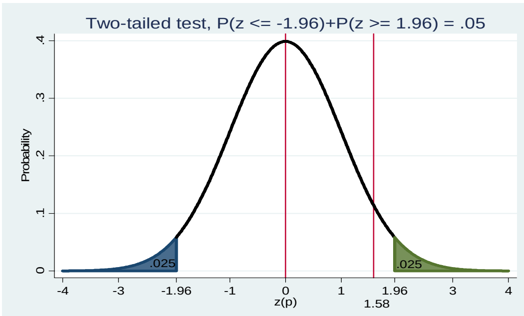
i. H_0: p=.15, H_a: μ≠.15 This is a two-tailed test. So, you have to consider both extremes.

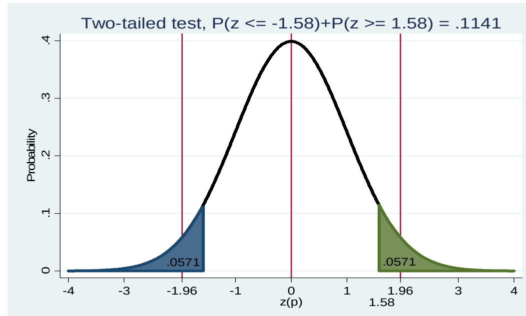
ii. *α=.05 (rejection region), the critical values are ±1.96.*

iii. $z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{.19 - .15}{\sqrt{.15 \times .85/200}} = 1.58$

iv. *Since TS (1.58) < CV (1.96),* do not reject the null hypothesis at the .05 level.

v. Therefore, the dropout rate for high school seniors is .15.





7.2 P-value approach:

i. H_0: p=.15, H_a: μ≠.15 This is a two-tailed test. So, you have to consider both extremes.

ii. *α=.05 (rejection region)*

iii. $z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{.19 - .15}{\sqrt{.15 \times .85/200}} = 1.58$, *p-value*

$P(z_p \le -1.58) + P(z_p \ge 1.58) = .1141$

iv. *Since p-value (.1141) > α (.05),* do not reject the null hypothesis at the .05 level. It is too risky (11 percent of being wrong) to reject the null hypothesis.

v. Therefore, the dropout rate for high school seniors is .15.

7.3 Confidence Interval approach:

i. H_0: p=.15, H_a: μ≠.15 This is a two-tailed test. So, you have to consider both extremes.

ii. *α=.05 (rejection region), 95 percent confidence interval, the critical values are ±1.96.*

iii. $\hat{p} \pm z_{\alpha/2} \frac{pq}{\sqrt{n}} = .19 \pm 1.96 \times \sqrt{\frac{.15 \times .85}{200}} = .19 \pm 0486$, *[.1414, .2386]*

iv. *The hypothesized proportion .15 exists between .1414 and .2386.* Therefore, we do not reject the null hypothesis at the .05 level.

v. The average cost of stroke rehabilitation is $24,672.