K300 (4392) Statistical Techniques (Fall 2007)

Assignment 1: Basic Math (Due September 5)

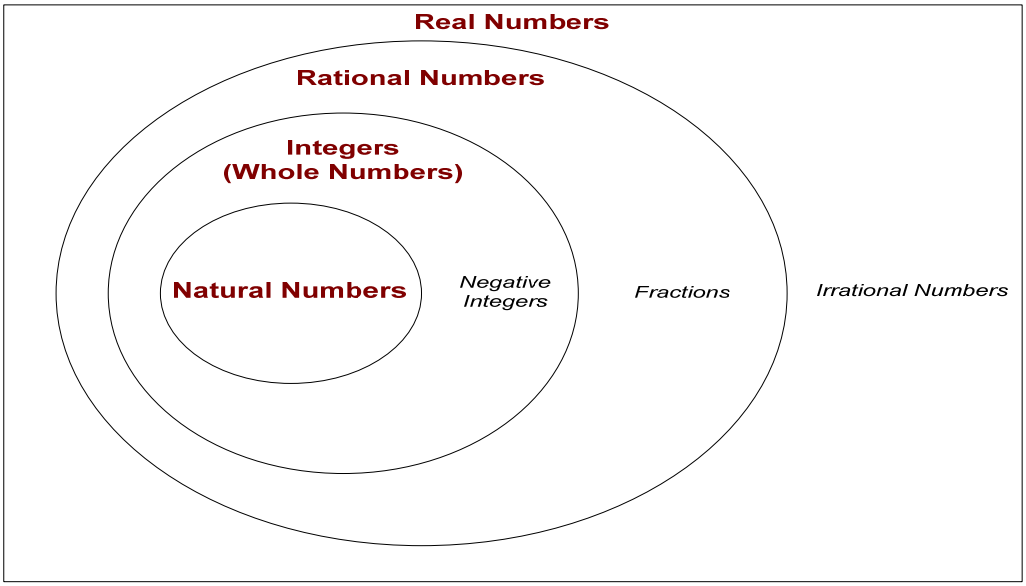
Name:_____ Points: _____ (110 points)

You need to understand basic algebra for this class. This assignment contains examples of key algebraic concepts and rules followed by questions you have to answer. Each question accounts for 4 points of the total. In order to do homework successfully, you

- MUST provide the appropriate steps to get the answers as shown in examples;
- work on this worksheet; DO NOT use separate sheets;
- return this worksheet on Wednesday, September 5.
- You MAY NOT discuss with other classmates when answering the questions.

If you have any problem with any of the questions, please post messages on Oncourse CL or come and see the instructor (kucc625@indiana.edu and (317) 274-0573).

Real Numbers



1. ***Natural numbers*** = {0, 1, 2, …} // nonnegative integers (0 and positive integers)
2. ***Integers*** = {… -2, -1, 0, 1, 2, …} // 0 plus positive and negative integers
3. ***Rational numbers*** include integers and fractions such as 1/2, -3/5, and 7/3.
4. ***Real numbers*** include rational numbers and irrational numbers such as $\sqrt{2}$

Part 1. Terminology

1. A ***variable*** is an unknown quantity that may change, while a ***constant*** is known and does not change at all. A variable `age` may have 20, 30, 50, and others depending on subjects. A constant π (pi) is the ratio of a circle's circumferences to its diameter, which is 3.141592…
2. A ***coefficient*** is a constant multiplicative fact of an object, preceding variables.

3. A ***term*** of equations consists of a variable and its coefficient. In 3x, for instance, x is a variable and 3 is its coefficient. The coefficient of 1 may be omitted.
4. Basic arithmetic operations: addition (+), subtraction (-), multiplication (×), and division (÷ or /). E.g., 5 × 3 = 5 + 5 + 5 = 15
5. a × 0 = 0; 0 ÷ a = 0; but division by zero (e.g., 5 ÷ 0) is not defined.
6. A ***square*** of a number is the number multiplied by itself. E.g., 3^2 is 3 × 3 = 9.
7. A ***square root*** of a number x is a number r such that $r^2 = x$. That is, $\sqrt{x} = r$.
8. **Question 1** (A short answer will do): x + 0 = x
9. **Question 2** (A short answer will do): x × 0 × 3y = 0

Part 2. Rules of Arithmetic Operations

1. ***Commutative law***: $a + b = b + a$; $a \times b = b \times a$
2. But $a - b \neq b - a$; $a \div b \neq b \div a$
3. ***Associative law***: $(a + b) + c = a + (b + c)$; $(ab)c = a(bc)$
4. ***Distributive law***: $(a + b)x = ax + bx$
5. $\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$
6. $\frac{a}{b} - \frac{c}{d} = \frac{ad - cb}{bd}$
7. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
8. $\frac{a}{b} \div \frac{c}{d} = \frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
9. Example: (3 + 4) + 5 = 3 + (4 + 5) = 3 + 9 = 12
10. Example: $(a + d)x + (b - c)y = ax + dx + by - cy$
11. Example: $\frac{2}{3} + \frac{5}{4} = \frac{2 \times 4 + 5 \times 3}{3 \times 4} = \frac{8 + 15}{12} = \frac{23}{12}$
12. Example: $\frac{7}{3} - 2 = \frac{7}{3} - \frac{2}{1} = \frac{7 \times 1 - 2 \times 3}{3 \times 1} = \frac{7 - 6}{3} = \frac{1}{3}$
13. Example: $\frac{2}{3} \times \frac{5}{4} = \frac{2 \times 5}{3 \times 4} = \frac{10}{12}$
14. Example: $\frac{2}{3} \div \frac{5}{4} = \frac{2/3}{5/4} = \frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$
15. **Question 3**: $\frac{2}{3} - \frac{5}{4} = \frac{2 \times 4 - 5 \times 3}{3 \times 4} = \frac{8 - 15}{12} = -\frac{7}{12}$
16. **Question 4**: $3 + \frac{2}{5} = \frac{3 \times 5 + 2}{5} = \frac{15 + 2}{5} = \frac{17}{5}$
17. **Question 5**: $\frac{2}{3} \times \frac{5}{4} = \frac{2 \times 5}{3 \times 4} = \frac{10}{12} = \frac{5}{6}$
18. **Question 6**: $\frac{3}{5} \div \frac{7}{4} = \frac{3/5}{7/4} = \frac{3}{5} \times \frac{4}{7} = \frac{3 \times 4}{5 \times 7} = \frac{12}{35}$

Part 3. Exponentiation I

1. $x^0 = 1,\ x^1 = x$ e.x., $9^0 = 1,\ 2^1 = 2$
2. $x \times x \times x... = x^n$ e.x., $3 \times 3 \times 3 \times 3 = 3^4$
3. $\frac{1}{x \times x \times x...} = \frac{1}{x^n} = x^{-n}$ e.x., $\frac{1}{7 \times 7 \times 7} = \frac{1}{7^3} = 7^{-3}$
4. $x^a \times x^b = [x \times x \times ...(a)][x \times x \times ...(b)] = x^{(a+b)}$
5. $\frac{x^a}{x^b} = \frac{x \times x \times x...(a)}{x \times x \times x...(b)} = x^a \times x^{-b} = x^{(a-b)}$
6. Example: $x^3 \times x^4 = [x \times x \times x][x \times x \times x \times x] = x^{(3+4)} = x^7$
7. Example: $\frac{5^7}{5^4} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = 5^7 \times 5^{-4} = 5^{(7-4)} = 5^3$
8. Example: $\frac{e^2 e^5}{e^4 e e^2} = \frac{[e \times e][e \times e \times e \times e \times e]}{[e \times e \times e \times e]e[e \times e]} = \frac{e^{(2+5)}}{e^{(4+1+2)}} = \frac{e^7}{e^7} = e^7 e^{-7} = e^{(7-7)} = e^0 = 1$
9. **Question 7**: $4^3 \times 4^2 = 4^{(3+2)} = 4^5$
10. **Question 8**: $\frac{x^3}{x^2} = x^3 \times x^{-2} = x^{(3-2)} = x$
11. **Question 9**: $\frac{y^2 y y^4}{y^5 y^3} = \frac{y^{(2+1+4)}}{y^{(5+3)}} = y^7 \times y^{-8} = y^{(7-8)} = y^{-1}$
12. **Question 10** (A short answer will do): $100^0 = 1$

Part 4. Exponentiation II

1. $[x^a]^b = x^a \times x^a ...(b) = x^{[a+a+...(b)]} = x^{(ab)}$
2. $\left(\frac{y}{x}\right)^a = \frac{y^a}{x^a}$
3. $\sqrt{x} = x^{1/2},\ \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-(1/2)}$
4. Example: $[5^3]^4 = 5^3 \times 5^3 \times 5^3 \times 5^3 = 5^{(3+3+3+3)} = 5^{(3\times4)} = 5^{12}$
5. Example: $[5^{-3} e^{1/2}]^2 = 5^{-3} \times 5^{-3} \times e^{1/2} \times e^{1/2} = 5^{(-3-3)} e^{(1/2+1/2)} = 5^{(-3\times2)} e^{(1/2\times2)} = 5^{-6} e$
6. Example: $\left(\frac{2^3}{3^4}\right)^2 = \frac{2^{(3\times2)}}{3^{(4\times2)}} = \frac{2^6}{3^8}$
7. Example: $e^3 \sqrt{e} = e^3 e^{1/2} = e^{3+1/2} = e^{7/2}$
8. Example: $3^2 \frac{1}{\sqrt{3}} = 3^2 3^{-1/2} = 3^{2-1/2} = 3^{3/2}$
9. **Question 11**: $[x^2]^3 = x^{2\times3} = x^6$
10. **Question 12**: $\left(\frac{e}{x^2}\right)^4 = \frac{e^4}{x^{(2\times4)}} = e^4 x^{-8}$
11. **Question 13**: $\sqrt{e} e^3 \sqrt{e} = e^{(1/2+3+1/2)} = e^4$

Part 5. Expansion and Factoring

1. $(a+b)^2 = (a+b)(a+b) = aa + ab + ab + bb = a^2 + 2ab + b^2$
2. $(a+b)(c+d) = (a+b)c + (a+b)d = ac + ad + bc + bd$
3. $(a+b)(a-b) = aa - ab + ab - bb = a^2 - b^2$
4. Example: $(3+x)^2 = (3+x)(3+x) = 3\times 3 + 3x + 3x + xx = 3^2 + 6x + x^2$
5. Example: $(1+2x)(3y-z) = (1+2x)3y + (1+2x)(-z) = 3y + 6xy - z - 2xz$
6. Example: $(3y-2)(3y+2) = (3y)(3y) + 2\times 3y - 2\times 3y - 2\times 2 = (3y)^2 - 2^2$
7. **Question 14**:
   $(2x+y)^2 = (2x+y)(2x+y) = (2x)^2 + 2xy + 2xy + y^2 = (2x)^2 + 4xy + y^2$
8. **Question 15**: $(3x-2z)(y+5) = 3xy + 3x\times 5 - 2yz - 2z\times 5 = 3xy + 15x - 2yz - 10z$
9. **Question 16**: $(x+3y)(x-3y) = x^2 - 3xy + 3xy - (3y)^2 = x^2 - (3y)^2$

Part 6. Summation

1. $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i = \sum x_i = x_1 + x_2 + ... + x_n$
2. $\sum_{i=1}^{n} a = a + a + ... = an$
3. $\sum_{i=1}^{n} ax_i = \sum (ax_i) = ax_1 + ax_2 + ax_3 + ... + ax_n = a\sum_{i=1}^{n} x_i$
4. $\sum_{i=1}^{n} (ax_i - by_i) = \sum (ax_i) - \sum (by_i) = a\sum x_i - b\sum y_i$
5. Example: $\sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5 = 15$
6. Example: $\sum_{i=1}^{7} 5 = 5 + 5 + 5 + 5 + 5 + 5 + 5 = 5\times 7 = 35$
7. Example: $\sum_{i=2}^{4} (x_i + 1) = (x_2 + 1) + (x_3 + 1) + (x_4 + 1) = x_2 + x_3 + x_4 + 3$
8. Example: $\sum_{i=1}^{3} 5x_i = 5x_1 + 5x_2 + 5x_3 = 5(x_1 + x_2 + x_3)$
9. **Question 17**: $\sum_{i=1}^{5} 3 = 3 + 3 + 3 + 3 + 3 = 3\times 5 = 15$
10. **Question 18**: $\sum_{i=5}^{7} 2x_i = 2x_5 + 2x_6 + 2x_7 = 2(x_5 + x_6 + x_7)$
11. **Question 19**: $\sum_{i=1}^{3} (2x_i - 1) = 2x_1 - 1 + 2x_2 - 1 + 2x_3 - 1 = 2(x_1 + x_2 + x_3) - 3$
12. **Question 20**: represent the summation of 5 through 100 using $\sum$. That is, 5 + 6 + 7 + 8 + … + 100. $\sum_{i=5}^{100} i$

Part 7. Solving Equations

1. Rearrange an equation so that only the unknown (what you want to know) is located on the left side of the equal sign. This task involves the inverse operation.
2. You may add or subtract the same quantity to the both sides of an equation.
3. You may multiply or divide the both sides of an equation by the same quantity.
4. Example:

By adding	By subtracting	By multiplying	By dividing
$x-3=2$ $x-3+3=2+3$ $x+0=5$ $x=5$	$2+x=5$ $2+x-2=5-2$ $0+x=3$ $x=3$	$\frac{x}{7}=4$ $\frac{x}{7}\times 7=4\times 7$ $x=28$	$8x=16$ $\frac{8x}{8}=\frac{16}{8}$ $x=2$

5. Example:

$2x-3=5$ $2x-3+3=5+3$ $2x=8$ $\frac{2x}{2}=\frac{8}{2}$ $x=4$	$3z=2x+z$ $3z-z=2x+z-z$ $2z=2x$ $2z\times\frac{1}{2}=2x\times\frac{1}{2}$ $z=x$ $x=z$	$\frac{2}{3x}=5$ $\frac{2}{3x}\times\frac{3}{2}=5\times\frac{3}{2}$ $\frac{1}{x}=\frac{15}{2}$ $x=\frac{2}{15}$	$4x-6=2x$ $4x-6+6=2x+6$ $4x=2x+6$ $4x-2x=2x+6-2x$ $2x=6$ $2x\times\frac{1}{2}=6\times\frac{1}{2}$ $x=3$

6. Questions (21-24): solve the equations with respect to *x*

Question 21	Question 22	Question 23	Question 24
$5+x=2$ $5+x-5=2-5$ $x=-3$	$2x-1=3$ $2x-1+1=3+1$ $2x=4$ $2x\times\frac{1}{2}=4\times\frac{1}{2}$ $x=2$	$\frac{x}{3}=2$ $\frac{x}{3}\times 3=2\times 3$ $x=6$	$3+3x=2x+4$ $3+3x-3=2x+4-3$ $3x=2x+1$ $3x-2x=2x+1-2x$ $x=1$

7. Questions (25-27): solve the equations with respect to *x*

Question 25	Question 26	Question 27 (6points)
$\frac{2}{3x}-5=3$ $\frac{2}{3x}-5+5=3+5$ $\frac{2}{3x}\times\frac{3}{2}=8\times\frac{3}{2}$ $\frac{1}{x}=12$, $x=\frac{1}{12}$	$2x-4z=6y-2$ $2x-4z+4z=6y-2+4z$ $2x=6y-2+4z$ $2x\times\frac{1}{2}=(6y-2+4z)\times\frac{1}{2}$ $x=3y-1+2z$	$6z-4y=-y+\frac{3x}{2}$ $6z-4y+y=-y+\frac{3x}{2}+y$ $6z-3y=\frac{3x}{2}$ $(6z-3y)\times\frac{2}{3}=\frac{3x}{2}\times\frac{2}{3}$ $4z-2y=x$, $x=4z-2y$